Model for estimating the refractive-index structure constant in clear-air intermittent turbulence

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We explain discrepancies in comparing estimations of the refractive-index structure constant C_n^2 in clear air by means of different techniques by taking into account atmospheric intermittency effects. We formulate a model of C_n^2 in intermittent turbulence on the basis of the Tatarskii theory, and we calculate the mean value of C_n^2 through a probabilistic approach. We deduce a factor, which gives a measure of the statistical reduction of turbulence that is due to intermittency, within the model framework. A procedure for estimating the mean value of C_n^2 from data of a specific radiosonde observation is illustrated.

Key words: Scintillation, refractive-index structure constant, turbulence intermittency.

1. Introduction

Scintillations are of relevant interest for both telecommunications and for remote sensing. On one hand, atmospheric turbulence can significantly affect the received field amplitude, degrading the signal-to-noise ratio[1]; on the other hand, measurement of the refractive-index fluctuation intensity can provide information on the local state of the atmosphere itself.[2] Clear-air scintillations are usually evaluated by means of refractive-index structure constant C_n^2, i.e., the amplitude of the spatial structure function of the refractive-index fluctuations between two points at unitary distance. The importance of the C_n^2 estimation is mostly related to the fact that it appears in the propagation equations of electromagnetic (e.m.) waves through random media.[3]

The main techniques we use to evaluate C_n^2 are based on the use of both e.m. remote sensors and meteorological radiosondes. The estimation of C_n^2 by means of radio and optical line-of-sight links, pulsed Doppler radars, and scintillometers is well documented in literature.[4–6] Alternatively we may derive C_n^2 from meteorological measurements that we make by different techniques, i.e., radiosonde observations (RAOB's) or special meteorological equipment.[7,8] The possibility of C_n^2 estimation from measured meteorological quantities may be relevant since it would permit a design optimization of electro-optical or radio systems that evaluates C_n^2 statistics in operative sites. Among the models based on meteorological measurements, the one proposed by Tatarskii[9] has been the most-used tool for calculating C_n^2 in clear air by means of measurements of the outer scale of turbulence and of the local temperature and humidity vertical gradients. However, in many cases, when we use radars or microwave satellite links, the e.m. estimations of C_n^2 in clear air appear appreciably lower than those we derive from the application of the Tatarskii model to contemporary RAOB data.[10,11] The fact that the values assumed for the outer scale are too large for the analyzed events[12] cannot justify the resultant disagreements between the C_n^2 values (up to 2 orders of magnitude). We may, however, explain the noted discrepancy by considering that the Tatarskii model applies to cases of homogeneous well-developed turbulence. Actually, in real atmosphere this condition is only seldom met because the clear air may be locally fluctuating between unstable and stable conditions, i.e., turbulence may be intermittent.[13,14] Intermittency effects are due to the variability of atmospheric parameters that are supposed to have large-scale fluctuations in addition to small-scale fluctuations, causing refractive-index structure constant C_n^2 to undergo random variations in both space and time.[15,16]

In this paper, in order to explain the disagreement between estimations of C_n^2 that are based on e.m.

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sensors and RAOB's, we develop a model of C_n^2 that takes turbulence intermittency as a random process into consideration. The formulation of the random model of C_n^2 basically involves both the Richardson number, to describe the local instability, and the Tatarskii theory of homogeneous turbulence. We calculate the mean value of C_n^2 with the aim of expressing the mean value of C_n^2 in intermittent turbulence through mean values of meteorological gradients, which are directly derivable from conventional RAOB's whose spatial resolution is generally much larger than the intermittency scales.[17] The scarcity of experimental data on small-scale distributions of atmospheric parameters has compelled us to assume analytical probability density functions (pdf's). Here we deduce a reduction factor, which was previously introduced in literature by some authors from experimental considerations,[16] under simplifying assumptions and interpret it as a measure of intermittency effects from meteorological data sets. As an example, for a specific summer RAOB, we show the estimation of the mean value of C_n^2 through spatial averages by using radiosonde measurements.

2. Model of C_n^2 in Intermittent Turbulence

Various models and, consequently, different techniques have been used so far to estimate the C_n^2 value from meteorological data in clear air.[7,8] The Tatarskii model of the microstructure of the refractive index in turbulent flow simply relates the value of C_n^2 to the outer scale of turbulence L_o and to the vertical gradient M of the refractivity.[9] Assuming a statistically stationary regime for a well-developed homogeneous turbulence following the Kolmogorov law, we give the refractive-index structure constant C_n^2 by[9]

$$C_n^2 = a^2 L_o^{4/3} M^2, \tag{1}$$

where $a^2 = 4.8$. If we denote the atmospheric buoyancy by $B = (g/\Theta)\mathrm{d}\Theta/\mathrm{d}z$, where g is the gravity acceleration, Θ is the potential temperature, and z is the altitude, and if we denote the vertical gradient of specific humidity q by $Q = \mathrm{d}q/\mathrm{d}z$, we may express vertical gradient M of the refractivity at microwaves (neglecting absorption and dispersion) in the following way:

$$M = \xi B + \zeta Q, \tag{2}$$

where

$$\xi = -77.6 \times 10^6 (p/Tg)(1 + 15{,}500 q/T), \tag{3a}$$

$$\zeta = -77.6 \times 10^6 (7800 p/T^2), \tag{3b}$$

where p, q, and T are local meteorological quantities.

The evaluation of C_n^2 by the use of Eq. (1) presents some inherent difficulties. Outer scale L_o exhibits values ranging from many meters to several tens of meters and, although profiles of L_o in the boundary layer have been reported in literature,[18] there is scarcity of experimental results. This fact has induced some authors to consider it an adjustable parameter[10] or has discouraged others from estimating C_n^2 by means of Eq. (1).[19]

In past years, measurements of clear-air turbulence by highly sensitive balloon-borne instrumentation have shown[20,21] that turbulence may be found in thin layers with sharp randomly varying boundaries. Also, this phenomenon of intermittency is strongly related to wind-shear instability.[22] However, if we use conventional RAOB's to estimate C_n^2 from Eq. (1), we find larger values compared with those derived by other sensors. In fact, the spatial resolution of radiosoundings (of the order of hundreds of meters) is such that the fine structure of the fluctuations of the meteorological parameters (which arises from intermittency) is washed out. Indeed, Eq. (1) does not take into consideration the intermittency effects that are due to random instabilities of local atmosphere. We have found evidence of these effects in diverse experiments by using microwave radio links[11] or e.m. or acoustic sensors.[23–25] The experimental results lead us to conclude that, in a thermally stratified atmosphere, thin horizontal turbulent layers are often embedded in large-scale laminar flows and are associated with small Richardson numbers; hence the need of a random model of C_n^2 in intermittent turbulence.

In a thermally stable atmosphere, turbulence develops when the buoyancy and the wind shear assume values in mutually conditioned ranges, which are characterized by the Richardson number. In fact the Richardson number R_i is an index of the local instability of the atmosphere and is defined as[3]

$$R_i = B/S, \tag{4}$$

where $S = |\mathrm{d}\mathbf{v}/\mathrm{d}z|^2$ is the square wind shear and $\mathbf{v}$ is the vectorial horizontal wind velocity. Only when R_i is less than or equal to the critical Richardson number R_{ic} is the stratification locally unstable and turbulence developed. The critical value R_{ic} has been shown to be equal to 0.25.[26] In intermittent turbulence the buoyancy and the wind shear undergo random fluctuations about their mean values over large scales, as does the Richardson number.[27] Note that within large scales the turbulence cannot be considered homogeneous, i.e., the intermittency effects are appreciable. If R_i is randomly fluctuating about the critical value R_{ic}, the local state of the atmosphere may pass from turbulence to stability in a random way, giving rise to intermittent or patched turbulence. Under these conditions, the use of the Tatarskii model should be limited only to regions in which $R_i \leq R_{ic}$.

Some considerations are needed in order to express the statistical dependence of C_n^2 on the meteorological variables. Equations (2) and (3) show that refractivity gradient M depends on local meterological quantities p, q, and T and on gradient quantities B and Q. However, we find that C_n^2 is mainly related to the structure constants of temperature and humidity, i.e., to their local vertical gradients.[8] If we suppose that local meterological quantities p, q, and T do not vary appreciably over large scales to affect the value of C_n^2 (see Ref. 28), we can assume from Eq. (2) that M is statistically dependent on only the gradient quantities, i.e., on buoyancy B and specific humidity gradient Q. In this case, we may suppose that quantities ξ and ζ in Eqs. (3a) and (3b) are constant over large scales, and we may replace them by ξ_0 and ζ_0, respectively. Also, we assume that L_o is randomly variable because of intermittent inhomogeneities and that in stable layers, i.e., where $R_i > R_{ic}$, the value of C_n^2 is 0. As a consequence, using Eqs. (1) and (4), we may suppose that refractive-index structure constant C_n^2 is a strongly nonlinear function of the random variables L_o, S, B, and Q and can be expressed by

$$C_n^2(L_o, S, B, Q) = a^2 L_o^{4/3}(\xi_0 B + \zeta_0 Q)^2 u(S - S_c), \quad (5)$$

where $u(S - S_c)$ is the step function centered on the critical square shear S_c, which is defined as $S_c = B/R_{ic}$. Equation (5) describes the intermittency effects on C_n^2 that are completely characterized by its pdf $p(C_n^2)$.

3. Mean Value of C_n^2 in Intermittent Turbulence

Denoting with angle brackets $\langle\,\rangle$ the ensemble average over large scales, we give the mean value $\langle C_n^2\rangle$ by

$$\langle C_n^2\rangle = \int_0^\infty C_n^2 p(C_n^2)\mathrm{d}C_n^2. \quad (6)$$

Various pdf's have been proposed to estimate the statistics of C_n^2, but they refer to data with spatial scales (of the order of hundreds of meters) that are much larger than the intermittency ones.[7] The scarcity of small-scale measurements of the statistical distribution of C_n^2 leads us to perform the integration in Eq. (6) in four-dimensional space (L_o, S, B, Q), i.e., in the space of the random variables by which C_n^2 is expressed in Eq. (5).[28,29] In fact, defining the joint pdf of L_o, S, B, and Q as $p(L_o, S, B, Q)$ and substituting Eq. (5) into Eq. (6) yields

$$\langle C_c^2\rangle = a^2 \int_{-\infty}^\infty \int_{-\infty}^\infty \int_{S_c}^\infty \int_0^\infty L_o^{4/3}(\xi_0 B + \zeta_0 Q)^2 \times p(L_o, S, B, Q)\mathrm{d}L_o\mathrm{d}S\mathrm{d}B\mathrm{d}Q. \quad (7)$$

We may simplify the calculations of the integrals in Eq. (7) by assuming (1) that refractivity gradient M is statistically independent of both the outer scale L_o and the square wind shear S, i.e., $p(L_o, S, B, Q) = p(L_o, S)p(B, Q)$. This means that the static parameters of instability (B and Q) are statistically uncoupled with respect to the dynamic parameters (L_o and S) in order to determine the intermittency effects[29]; (2) that the value of S_c is a constant given by $S_c = \langle B\rangle/R_{ic}$, since the relative fluctuations of B about its large-scale mean value are often small with respect to those of S in free atmosphere[22,27]; (3) that the correlation coefficient between B and Q, denoted by r_{BQ}, is equal to ± 1, since there is experimental evidence that the correlation between the fluctuations of Q and B over large scales is often high (close to 1), positive or negative.[28]

Using the above hypotheses and expressing the double integral in B and Q in terms of the variances σ_B^2 and σ_Q^2 of B and Q, respectively, and of the correlation coefficient r_{BQ},[29] we reduce Eq. (7) to

$$\langle C_n^2\rangle = a^2 F \langle L_o\rangle^{4/3}\langle M\rangle^2, \quad (8)$$

where

$$F = \left[(1 + \sigma_M^2/\langle M\rangle^2) \times \int_{S_c}^\infty \int_0^\infty L_o^{4/3} p(L_o, S)\mathrm{d}L_o\mathrm{d}S\right] \Big/ \langle L_o\rangle^{4/3}, \quad (9)$$

and $\langle M\rangle$ and σ_M^2 are the mean value and the variance of refractivity gradient M, respectively. These are given by:

$$\langle M\rangle^2 = (\xi_0\langle B\rangle + \zeta_0\langle Q\rangle)^2, \quad (10)$$

$$\sigma_M^2 = (\xi_0\sigma_B \pm \zeta_0\sigma_Q)^2, \quad (11)$$

where σ_B and σ_Q are the standard deviations of B and Q, respectively, and the sign $\pm$ of the term $\zeta_0\sigma_Q$ corresponds to the sign of the unitary correlation coefficient r_{BQ}.

Equation (8) shows the similarity of the $\langle C_n^2\rangle$ expression with the Tatarskii formula given in Eq. (1). The factor F gives a measure of the intermittency effects on C_n^2 values, but its evaluation is not easily performed because of the difficulties of assigning realistic expressions to $p(L_o, S)$ and σ_M^2. Further approximations are therefore needed. If we can assume statistical independence between L_o and S, i.e., $p(L_o, S) = p(L_o)p(S)$, we can reduce the factor F in Eq. (9) to

$$F = (1 + \sigma_M^2/\langle M\rangle^2)F_S F_{L_o}, \quad (12)$$

where

$$F_S = \int_{S_c}^\infty p(S)\mathrm{d}S, \quad (13)$$

$$F_{L_o} = \left[\int_0^\infty L_o^{4/3} p(L_o)\mathrm{d}L_o\right] \Big/ \langle L_o\rangle^{4/3}. \quad (14)$$

Even though there are no experimental results on the statistical independence between L_o and S, Eq. (12) allows us to emphasize the different contributions of the meteorological parameters to intermittency. The scarce availability of small-scale observations still gives rise to difficulties in choosing $p(L_o)$ and $p(S)$. However, according to Ref. 28, the form of a pdf is less important than accurate estimates of its parameters, which are given in terms of the observed large-scale data. Thus, in general, the simplest pdf form consistent with measurements may be choosen. Diverse pdf's may be assumed for the shear $\sqrt{S}$ (or for the square shear S). If the horizontal components of the shear vector are supposedly normally distributed with the same standard deviation, the shear $\sqrt{S}$ results that are distributed according to the Rice–Nagakami pdf and the factor F_S may be expressed by

$$F_S(\langle\sqrt{S}\rangle, \sigma_{\sqrt{S}}, \sqrt{S_c}) = \int_{-\infty}^{-\sqrt{S_c}} p(\sqrt{S})\mathrm{d}\sqrt{S} + \int_{\sqrt{S_c}}^{\infty} p(\sqrt{S})\mathrm{d}\sqrt{S}, \quad (15)$$

where

$$p(\sqrt{S}) = (\sqrt{S}/\sigma_{\sqrt{S}}^2)\exp[-(S + \langle S\rangle)/2\sigma_{\sqrt{S}}^2] \times I_0(\sqrt{S}\,\langle\sqrt{S}\rangle/\sigma_{\sqrt{S}}^2), \quad (15a)$$

I_0 is the first-order modified Bessel function, and $\sigma_{\sqrt{S}}^2$ is the variance of $\sqrt{S}$. Figure 1 shows the factor F_S calculated from Eq. (15) as a function of the mean shear $\langle\sqrt{S}\rangle$, normalized to the critical shear $\sqrt{S_c}$, and parametrized to the ratio $\sigma_{\sqrt{S}}/\langle S\rangle$. It is clear how, even for $\langle\sqrt{S}\rangle/\sqrt{S_c} < 1$, some turbulence is present and F_S is not 0. Appreciable reduction may also be found when $\langle\sqrt{S}\rangle/\sqrt{S_c} > 1$. If $\sigma_{\sqrt{S}} = 0$, no shear fluctuation is present, and F_S corresponds to the step function $u(\langle\sqrt{S}\rangle/\sqrt{S_c} - 1)$. The same behavior of the factor F_S has been shown to be used for a shear, a Gaussian, or a log-normal pdf.[29] From radar measurements, values of F_S, which were introduced empirically as a reduction factor, ranging from 0.1 in troposphere to 0.01 in stratosphere, have often been found.[10]

We may evaluate the factor F_{L_o}, given in Eq. (14), assuming, for instance, a uniform value for $p(L_o)$ between L_{omin} and L_{omax}, which are the minimum and the maximum values experimentally found for L_o, respectively. In this case, Eq. (14) for F_{L_o} yields

$$F_{L_o}(L_{omin}, L_{omax}) = 1.08({L_{omax}}^{7/3} - {L_{omin}}^{7/3})/(L_{omax} - L_{omin})^{7/3}, \quad (16)$$

where we may assume that L_{omin} ranges from 0.1 to 5 m and L_{omax} ranges from 30 to 100 m.[20,22] Figure 2 shows the factor F_{L_o} as a function of L_{omax}, with the result that $F_{L_o} < 1$ even for $L_{omax} = 100$ m. In the same figure we plotted the curve of an effective value L_{oeff} of the outer scale, which is defined as

$$L_{oeff} = F_{L_o}\langle L_o\rangle^{4/3} \quad (17)$$

The quantity L_{oeff} may be useful for applications since it depends only on the knowledge of the variability range of the outer scale L_o within the region under observation.

Finally, it is worth noting that, if there is no intermittency and the turbulence is well developed and homogeneous, then $\sigma_M^2 = 0$, $p(S) = 0$ for $S < S_c$, and L_o is a constant. In this case, as expected, factor F reduces to 1 and Eq. (8) becomes equal to the Tatarskii expression given in Eq. (1).

4. Example of the Estimation of the Mean Value of C_n^2

The applicability of the expressions given in Eqs. (8) and (12) requires knowledge of the variability range of L_o, the mean values, and the standard deviations of S, B, and Q from small-scale meteorological data sets. But these data are generally not available because the larger amount of atmospheric measurements are mostly derived from RAOB's with rough vertical resolutions of the order of hundreds of meters. However, an estimate of $\langle C_n^2\rangle$ may be possible through the following operative procedure. If we consider a slab of atmosphere that is defined by two consecutive RAOB measurements, we may approximate the mean

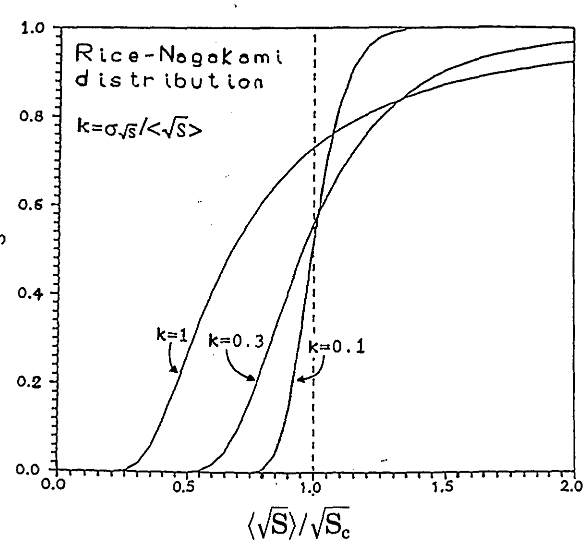

Fig. 1. Factor F_S as function of the ratio $\langle\sqrt{S}\rangle/\sqrt{S_c}$ parametrized to the ratio $k = \sigma_{\sqrt{S}}/\langle\sqrt{S}\rangle$, where a Rice–Nagakami pdf for the local wind shear $\sqrt{S}$ is assumed.

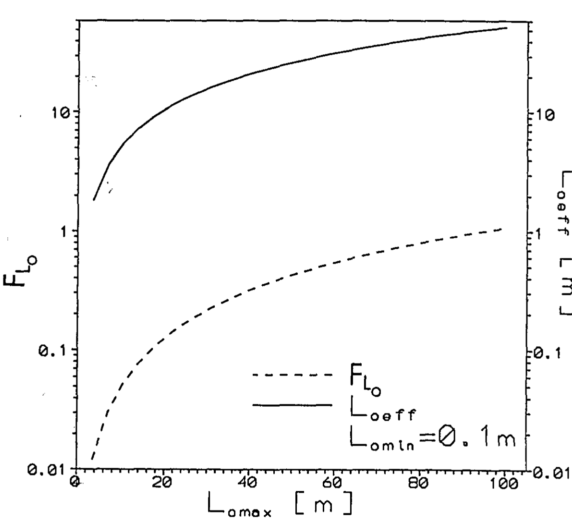

Fig. 2. Factor F_{L_o} and effective outer scale L_{oeff} as functions of the maximum outer scale L_{omax} when a uniform pdf for L_o and a minimum outer scale $L_{omin} = 0.1$ m is assumed.

values of meteorological parameters over large scales through their spatial averages within the slab. Furthermore, the fluctuations of M and S about their spatial averages are generally small andmay be expressed in terms of the spatial averages themselves,[30] as is shown below. This means that, for a specific RAOB, we can carry out an estimation of $\langle C_n^2 \rangle$ by evaluating the mean values of meteorological variables and their gradients through the spatial averages of RAOB data. In this context, $p(C_n^2)$ and, consequently, $p(L_o, S, B, Q)$, assumes the meaning of the probability density of occurrence of a given turbulence within the considered slab, and $\langle C_n^2 \rangle$ is intended as a slab spatial average.

We have examined more than thirty RAOB's performed in various sites in Italy under various meteorological conditions and in all the seasons. As an example of calculation, we refer to a summer RAOB with a vertical spatial resolution of ~ 350 m or smaller, which was performed in Cagliari–Elmas, Italy, by the Servizio Meteorologico Aeronautica Militare Italiana (S.M.A.M.I.) on 3 August 1976. Figure 3 shows the profiles of temperature T, specific humidity q, and Richardson number R_i. Note that the high positive values of R_i for the slab near 3500 m appear stable. Nevertheless intermittency in the fine structure of turbulence is not excluded. Only an examination of the corresponding value of the factor F can give an appreciation of intermittency intensity. In the following we apply the procedure described above for estimating factor F from RAOB data.

First, it is shown that the ratio $\sigma_M^2/\langle M \rangle^2$ is considerably less than 1 and can then be neglected in Eq. (12). In fact, Fig. 4 shows the upper bound and the lower bound ($r_{BQ} = +1$ and $r_{BQ} = -1$, respectively) of the vertical profile of $\sigma_M^2/\langle M \rangle^2$, which is given by the ratio between Eqs. (11) and (10). The standard deviation σ_B is related to the mean air density ρ, to the effective

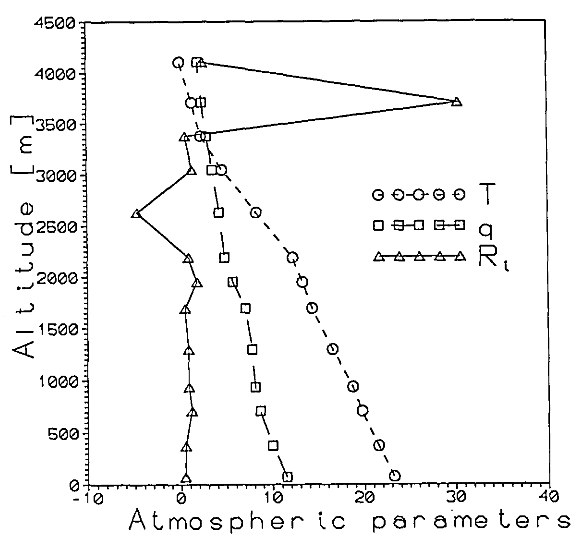

Fig. 3. Vertical profiles of temperature T (degrees Centigrade), specific humidity q, and Richardson number R_i in the lower troposphere, which we derived from RAOB data measured by the S.M.A.M.I. in Cagliari-Elmas, Italy, on 3 August 1976.

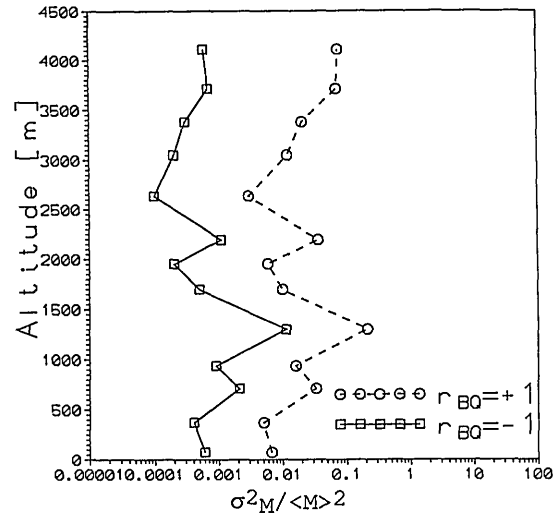

Fig. 4. Vertical profiles of the ratio $\sigma_M^2/\langle M \rangle^2$ for the same RAOB of Fig. 3. We assume a correlation coefficient $r_{BQ} = +1$ and $r_{BQ} = -1$ and use the expression given in Eq. (18) for σ_B^2 and a value proportional to σ_B^2 for σ_Q^2.

outer scale $L_{0\mathrm{eff}}$, and to the mean buoyancy by an empirical formula found in Ref. 30 and is supposed to be valid for the whole troposphere:

$$\sigma_B = 0.19\, L_{0\mathrm{eff}}^{-0.3} \langle B \rangle^{0.75} \langle \rho \rangle^{-0.15}, \qquad (18)$$

where it is assumed that $L_{0\max} = 20$ m and $L_{0\min} = 0.1$ m, which results in $L_{0\mathrm{eff}} = 10$ m. Since there are no known references for analogous expressions of σ_Q^2, we have assumed in the calculations that σ_Q^2 is one third of σ_B^2. Examining the figure, we can see that at each altitude σ_M^2 is much smaller than $\langle M \rangle^2$, and the resultant $\sigma_M^2/\langle M \rangle^2$ is no greater than 0.2. It is interesting to note that at optical frequencies at which the terms of refractivity that depend on humidity are negligible,[8] the same approximation may be assumed. In this case, remembering from Eq. (2a) that ξ_0 is assumed to be constant within the slab, we may approximate Eqs. (10) and (11) by

$$\langle M \rangle_{\mathrm{dry}}^2 = [-77.6 \times 10^6 (\langle p \rangle / \langle T \rangle g) \langle B \rangle]^2, \qquad (19a)$$

$$\sigma_{M\mathrm{dry}}^2 = [-77.6 \times 10^6 (\langle p \rangle / \langle T \rangle g) \sigma_B]^2. \qquad (19b)$$

Figure 5 shows, for the radiosounding as Fig. 3, the profiles of the ratios $\sigma_{M\mathrm{dry}}^2/\langle M \rangle_{\mathrm{dry}}^2$, which, in this case, still result in the variance of M being generally negligible with respect to its mean value. We have also found the above results by analyzing all the available RAOB data set.

Thus we can assume that, in a wide range of frequencies and at various altitudes, $\langle M \rangle^2 > \sigma_M^2$ and the intermittency factor reduces to $F \cong F_S F_{L_o}$. This means that the observed reduction in the C_n^2 value can be attributed mainly to the wind shear and to the outer scale fluctuations within the slab, while the buoyancy and the humidity effects are negligible in most cases.

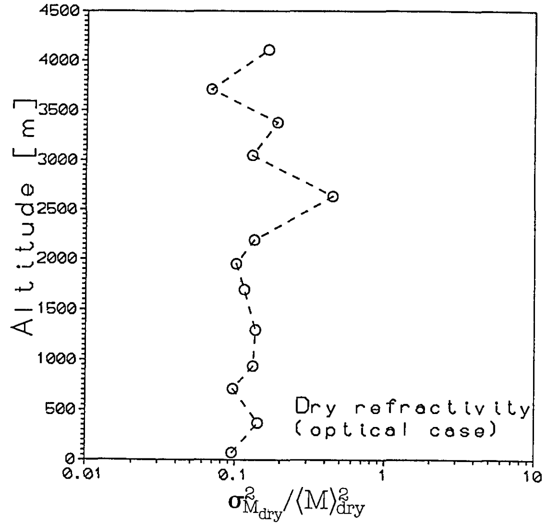

Fig. 5. Vertical profiles of the ratios $\sigma_{M\text{dry}}^2/\langle M\rangle_{\text{dry}}^2$ for the same RAOB as Fig. 3, for σ_b^2 the expression given in Eq. (18).

In order to derive the vertical profile of the factor F_S, we must also know the standard deviation $\sigma_{\sqrt{S}}$ of the shear. At each altitude we have calculated $\sigma_{\sqrt{S}}$ from RAOB data by applying the following empirical relationship[30]:

$$\sigma_{\sqrt{S}} = 0.18\, L_{\text{oeff}}^{-0.3}\langle B\rangle^{0.25}\langle\rho\rangle^{-0.15}. \tag{20}$$

If we assume the Rice–Nagakami pdf for the shear $\sqrt{S}$, we can derive the value of F_S directly from Fig. 1. The profile of F_S is plotted in Fig. 6 together with the ratio $\sigma_{\sqrt{S}}/\langle\sqrt{S}\rangle$, which is the parameter of the curves in Fig. 1. It is worth noting that F_S reaches its minimum value for the slab near 3500 m, (where R_i has its maximum), and that F_S is no greater than 0.3 along the vertical profile.

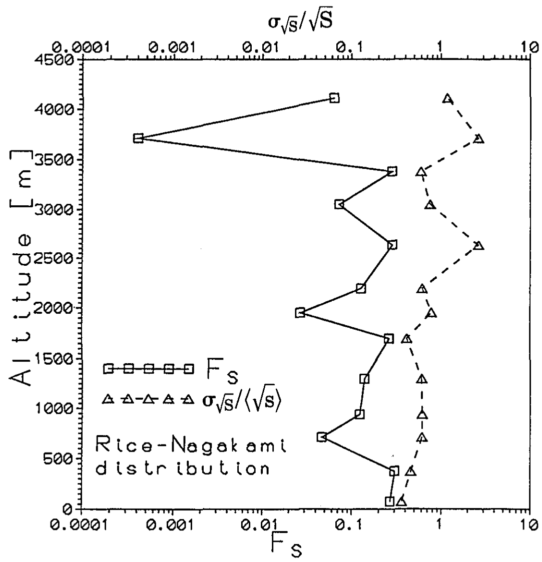

Fig. 6. Vertical profile of the factor F_S and of the ratio $\sigma_{\sqrt{S}}/\langle\sqrt{S}\rangle$ for the same RAOB of Fig. 3, assuming the expression given in Eq. (20) for $\sigma_{\sqrt{S}}$.

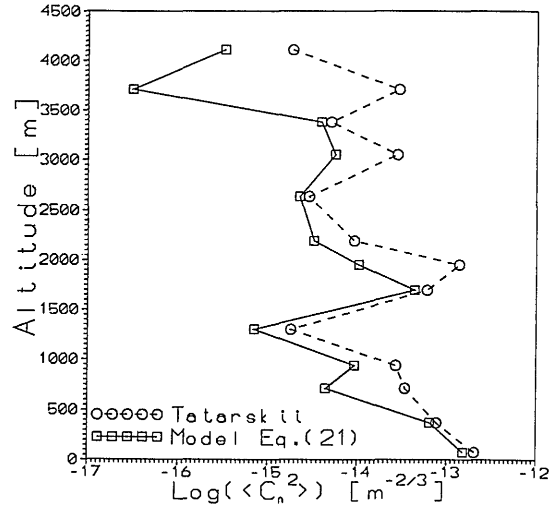

Fig. 7. Mean value of C_n^2 derived from the application of the Tatarskii formula given in Eq. (1) (dashed curve) and of the simplified model given in expression (21) (solid curve) to the RAOB data of Fig. 3.

As a consequence of the previous results, and substituting Eq. (17) into Eq. (8), we may estimate $\langle C_n^2\rangle$ from a specific RAOB by

$$\langle C_n^2\rangle \cong a^2 F_S L_{\text{oeff}}^{4/3}\langle M\rangle^2 \tag{21}$$

Figure 7 shows the estimated profiles of C_n^2 that we calculated from expression (21) and by applying the Tatarskii formula given in Eq. (1) directly to each slab data. As expected, the intermittency effects included in expression (21) give rise to a reduction in the value of C_n^2 with respect to the corresponding C_n^2 values derived from Eq. (1). This reduction of the values of C_n^2 depends on altitude; the highest difference occurs for the noted slab near 3500 m.

5. Conclusions

The atmospheric intermittency effects can affect the estimation of C_n^2 in clear air in a significant way. It has been shown that in intermittent turbulence the C_n^2 values are appreciably reduced with respect to the C_n^2 values in homogeneous turbulence. The random model of C_n^2, which is based on the Tatarskii theory, takes into account the intermittency in free atmosphere, and expressions for the calculation of the mean value of the C_n^2 have been deduced under simplifying assumptions that lead to expression (21). We may estimate the mean value of C_n^2 for a specific event by deriving the needed quantities from RAOB measurements. Knowledge of the variability range of the outer scale at each height would provide the value of the effective outer scale, even though there are no relationships to extract its vertical profile from RAOB data. The observed reduction in the C_n^2 value can be mostly attributed to wind-shear random variations, while the buoyancy and humidity effects are negligible in most cases. Wind-shear reduction factor F_S, in its simplest form, is independent of frequency. The value of F_S, ranging from 0 to 1, may be interpreted as a measure of intermittency effects on clear-air turbulence derived from RAOB data sets.

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