Comparison of Advanced Radar Polarimetric Techniques for Operational Attenuation Correction at C Band

GIANFRANCO VULPIANI, PIERRE TABARY, AND JACQUES PARENT DU CHATELET
Direction des Systèmes d’Observation, Météo-France, Trappes, France

FRANK S. MARZANO
Department of Electronic Engineering, University La Sapienza, Rome, Italy

(Manuscript received 31 October 2006, in final form 11 June 2007)

ABSTRACT

Rain path attenuation correction is a challenging task for quantitative use of weather radar measurements at frequencies higher than S band. The proportionality relationship between specific attenuation \( \sigma_{hh} \) and specific differential phase \( K_{dp} \) is the basis for simple path-integrated attenuation correction using differential phase \( \Phi_{dp} \). However, the coefficients of proportionality are known to be dependent upon temperature, on the one hand, and shape and raindrop size distribution, on the other hand. To solve this problem, a Bayesian classification scheme is proposed to empirically find the prevailing rain regime and adapt the \( \Phi_{dp} \)-based method. The proposed approach herein is compared with other polarimetric techniques currently available in the literature. Several episodes observed in the Paris, France, area by the C-band dual-polarized weather radar operating in Trappes (France) are analyzed and results are discussed.

1. Introduction

Quantitative use of weather radar measurements (i.e., precipitation estimation, hydrometeor classification) requires accurate evaluation of the main error sources, such as calibration, ground clutter, anomalous propagation, beam blockage, and rain path attenuation (e.g., Bringi and Chandrasekar 2001). At frequencies higher than S band, the latter becomes significant and needs to be compensated. Since the beginning of radar meteorology, several approaches for attenuation correction have been suggested in the literature (Hitschfeld and Bordan 1954; Meneghini et al. 1983; Bringi et al. 1990; Testud et al. 2001; Vulpiani et al. 2005, and many others). To overcome the instability of the iterative approaches (Hildebrand 1978; Aydin et al. 1989) that occur when path-integrated attenuation (PIA) is relatively large, the use of total path-integrated attenuation as a constraint has been first proposed for lidar applications (Klett 1981), complementing the Hitschfeld–Bordan (HB) method (Hitschfeld and Bordan 1954). Following the same approach the so-called surface reference techniques (SRTs) have been introduced for spaceborne weather radar (Meneghini et al. 1983; Iguchi and Meneghini 1994; Marzoug and Amayenc 1994). Dual-polarized radars can provide an estimation of path-integrated attenuation using the total differential propagation phase because specific attenuation and differential attenuation (\( \sigma_{hh} \) and \( \alpha_{dp} \)) are almost linearly related to specific differential phase \( K_{dp} \) at typical ground-based radar frequency bands (S, C, and X) (Bringi et al. 1990). The linearity of the relationship has been empirically verified by Gourley et al. (2007). Based on this evidence, the use of the differential phase constraint to correct the observed reflectivity and differential reflectivity was proposed and evaluated in Testud et al. (2000) and Le Bouar et al. (2001), respectively. A self-consistent scheme was proposed by Bringi et al. (2001) in order to account for the sensitivity of the proportionality factor between \( \sigma_{hh} \) (\( \alpha_{dp} \)) and \( K_{dp} \) to both temperature and drop size distribution. Recently, the same constraint has been used to estimate the intrinsic reflectivity and differential reflectivity at

Corresponding author address: Gianfranco Vulpiani, Italian Department of Civil Protection, Via Vitorchiano 4, 00189 Rome, Italy.
E-mail: gianfranco.vulpiani@protezionecivile.it

DOI: 10.1175/2007JTECHA936.1

© 2008 American Meteorological Society
the last range bin (the farthest from the radar) retrieving, through an iterative scheme, the range profiles of specific attenuation and differential attenuation (Vulpiani et al. 2005).

The aim of this work is to propose an improved version of the standard procedure for attenuation correction based on the use of differential phase shift (Bringi et al. 1990). A preliminary classification of the prevailing rain regime enables the use of adaptive relationships between specific attenuation and specific differential phase shift. The comparison with other advanced polarimetric techniques is accomplished on several cases observed during 2005–06 in the Paris, France, area by the C-band dual-polarized weather radar operating in Trappes, France (Gourley et al. 2006a).

2. Theoretical background

A gamma raindrop size distribution (RSD), with the general form \( N(D) = N_0 D^\mu \exp(-\Lambda D) \), where \( D \) is the particle diameter and \( N_0, \Lambda, \) and \( \mu \) are RSD parameters, has been introduced in the literature to account for most of the variability occurring in the naturally observed RSD. The concept of normalization has been introduced by Willis (1984) and revisited by Chandrasekar and Bringi (1987) and Testud et al. (2001). The number of raindrops per unit volume per unit size can be written as

\[
N(D) = N_w f(\mu) \left( \frac{D}{D_0} \right)^\mu \exp\left[ -(3.67 + \mu) \frac{D}{D_0} \right],
\]

where \( f(\mu) \) is a function \( \mu \) only, the parameter \( D_0 \) is the median volume drop diameter, \( \mu \) is the shape parameter of the drop spectrum, and \( N_w \) (mm\(^{-1}\) m\(^{-3}\)) is a normalized drop concentration that can be calculated as a function of liquid water content \( W \) and \( D_0 \) (e.g., Bringi and Chandrasekar 2001).

The copolar equivalent radar reflectivity factors \( Z_{hh} \) and \( Z_{vv} \) (mm\(^6\) m\(^{-3}\)) at horizontal (\( h \)) and vertical (\( v \)) polarization state can be expressed as follows:

\[
Z_{hh,sv} = \frac{\lambda^4}{\pi^2 |K|^2} \int_{D_{min}}^{D_{max}} 4\pi |f_{hh}(D)|^2 N(D) \, dD,
\]

where \( D_{min} \) and \( D_{max} \) are the minimum and maximum drop diameters, respectively, \( S_{hh,sv} \) is the backscattering copolar component of the scattering amplitude matrix \( S \) (mm\(^2\)) at \( h \) and \( v \) polarization, respectively, \( \lambda \) is the wavelength, and \( K \) is a constant defined as \( K = (\epsilon - 1)/(\epsilon + 2) \), where \( \epsilon \) is the complex dielectric constant of water estimated as a function of wavelength and temperature (Ray 1972).

Differential reflectivity \( Z_{dr} \) is defined as the ratio of reflectivity factors at two orthogonal polarizations, that is,

\[
Z_{dr} = \frac{Z_{hh}}{Z_{sv}}. \tag{3}
\]

The specific differential phase shift \( K_{dp} \) (\( \circ \) km\(^{-1}\)), which is due to the propagation phase difference between the two orthogonal polarizations and the specific power attenuation \( \alpha_{hv} \) (dB km\(^{-1}\)), can be obtained in terms of the forward copolar scattering amplitudes \( f_{hh} \) and \( f_{sv} \) as

\[
K_{dp} = 10^{-3} \frac{180}{\pi} \lambda \text{ Re} \int_{D_{min}}^{D_{max}} \left[ f_{hh}(D) - f_{sv}(D) \right] N(D) \, dD, \tag{4}
\]

\[
\alpha_{hv} = 8.686 \times 10^{-3} \lambda \text{ Im} \int_{D_{min}}^{D_{max}} f_{hh,sv}(D) N(D) \, dD, \tag{5}
\]

where \( \lambda \) is in meters. The specific differential attenuation \( \alpha_{dp} \) (dB km\(^{-1}\)) is obtained as \( \alpha_{dp} = \alpha_{hv} - \alpha_{sv} \). The copolar correlation coefficient \( \rho_{hv} \) is defined as

\[
\rho_{hv} = \frac{\int_{D_{min}}^{D_{max}} S_{sv}(D) S_{hh}^*(D) N(D) \, dD}{\sqrt{\int_{D_{min}}^{D_{max}} |S_{hh}(D)|^2 N(D) \, dD \int_{D_{min}}^{D_{max}} |S_{sv}(D)|^2 N(D) \, dD}} = |\delta_{hv}| e^{i\delta_{hv}}, \tag{6}
\]

where \( \delta_{hv} \) (\( \circ \)) is the backscattering differential phase shift.

When attenuation occurs, the measured reflectivity at horizontal polarization \( Z_{hh} \) (mm\(^6\) m\(^{-3}\)) and differential reflectivity \( Z_{dr} \) (in linear units) can be written as

\[
Z_{hh}'(r) = Z_{hh}(r) \exp \left[ -0.46 \int_0^r \alpha_{hh}(s) \, ds \right],
\]

\[
Z_{dr}'(r) = Z_{dr}(r) \exp \left[ -0.46 \int_0^r \alpha_{dp}(s) \, ds \right]. \tag{7}
\]
3. Advanced polarimetric techniques for attenuation correction

As scattering simulations have demonstrated (Bringi et al. 1990; Jameson 1992), specific attenuation \([\alpha_{hh} \text{ (dB km}^{-1}\text{)}]\) and differential attenuation \([\alpha_{dp} \text{ (dB km}^{-1}\text{)}]\) are almost linearly related to specific differential phase when \(D_0 \leq 2.5 \text{ mm}\) (where \(D_0\) is the median volume drop diameter),

\[
\begin{align*}
\alpha_{hh} &= \gamma_{hh} K_{dp}, \\
\alpha_{dp} &= \gamma_{dp} K_{dp},
\end{align*}
\]

where \(\gamma_{hh,dp}\) depends on drop size distribution, drop shape (equivalently on axis ratio), and temperature (Bringi and Chandrasekar 2001). Starting from this paradigm, a simple attenuation correction technique (hereafter PDP), based on the relation between differential phase \(\Phi_{dp}\) and cumulative attenuation \(A_{hh}\) and cumulative differential attenuation \(A_{dp}\), respectively, has been proposed by Bringi et al. (1990) and evaluated by other authors (i.e., Carey et al. 2000; Gourley et al. 2007). Assuming \(\gamma_{hh}\) is constant in range, \(A_{hh}\) (dB) can be expressed as

\[
A_{hh}(r) = \int_{r_0}^{r} \alpha_{hh}(s) \, ds,
\]

\[
= \gamma_{hh} \int_{r_0}^{r} K_{dp}(s) \, ds,
\]

\[
= \frac{\gamma_{hh}}{2} [\Phi_{dp}(r) - \Phi_{dp}(r_0)],
\]

\[
= \frac{\gamma_{hh}}{2} \Delta \Phi_{dp}(r, r_0).
\]

Consequently, the corrected reflectivity becomes

\[
10 \log_{10}[Z_{hh}(r)] = 10 \log_{10}[Z'_{hh}(r)] + 2A_{hh}(r)
\]

\[
= 10 \log_{10}[Z'_{hh}(r)]
\]

\[
+ \gamma_{hh} \Delta \Phi_{dp}(r, r_0).
\]

Equations analogous to (9) and (10) can be derived for \(A_{dp}\) and \(Z_{dr}\), respectively.

Attenuation correction techniques that assume constant values for \(\gamma_{hh,dp}\) can be in error due to both temperature variations as well as variations in shape (Jameson 1992). Many authors have noted that both the attenuation and differential attenuation due to “giant” raindrops along the propagation path result in values of \(\gamma_{hh,dp}\) that are nearly twice the theoretical values expected from scattering simulations (Smyth and Illingworth 1998; Carey et al. 2000; Gourley et al. 2006a).

a. Rain profiling algorithms

In the present study we will compare the performance of two techniques for attenuation correction belonging to the rain profiling algorithms. These approaches constitute a family of analytical solutions of the differential equation obtained from (7) when a power-law relation is assumed between \(\alpha_{hh}\) and \(Z_{hh}\) \((\alpha_{dp} = aZ_{hh}^b)\), with the estimated PIA being assumed as a boundary condition (Marzoug and Amayenc 1994; Iguchi and Meneghini 1994; Vulpiani et al. 2006). A detailed derivation thereof is presented in the appendix. Using the notation introduced in (9), the PIA can be written as \(A_{hh}(r_n)\), with \(r_n\) being the range distance at which the rain cell ends. Apart from the final value (FV) method, rain profiling algorithms also include the so-called constant adjustment (CA). The CA, which assumes that \(a\) (or equivalently \(N_w\)) is constant in range, adjusts the solution for the radar constant, resulting in a solution that is independent from the calibration error. The so-called ZPHI algorithm, proposed in Testud et al. (2000), represents the polarimetric version of the rain profiling CA algorithm that makes use of the differential phase shift to estimate \(A_{hh}(r_n)\) [by means of (9)]. A self-consistent scheme to improve the ZPHI method, taking into account the temperature and shape dependency of \(\gamma_{hh,dp}\), was proposed by Bringi et al. (2001). In the present work, we focus on the polarimetric versions of FV (PFV) and CA (PCA) using the \(\Phi_{dp}\), constraint to estimate the PIA.

b. Temperature and size distribution dependency of \(\gamma_{hh,dp}\)

The error structure of PDP and ZPHI was recently analyzed in a simulated framework by Gorgucci and Chandrasekar (2005). PDP is mainly conditioned by errors in the measured profiles of \(\Phi_{dp}\), while PCA might be not accurate in the presence of highly variable drop size distribution, with \(N_w\) having been assumed to be constant in range. The authors concluded that, generally, the algorithms provide comparable results in terms of cumulative attenuation estimation.

Algorithms making use of differential phase to estimate cumulative attenuation are sensitive to the assumed slope parameters in (8). As a matter of fact, \(\gamma_{hh,dp}\) is both temperature and size dependent (Jameson 1992). For small-to-medium drop sizes, temperature is the parameter that rules the \(\gamma_{hh,dp}\) variability, with molecular absorption dominating attenuation. At C-band frequencies, when raindrop size increases scattering tends to overwhelm absorption. For drop diameters...
larger than 5 mm, resonance scattering effects occur and size becomes the main reason for the variability of $\gamma_{hh,dp}$ (Ryzhkov et al. 2006). In Carey et al. (2000), the authors suggested determining the optimal values of $\gamma_{hh}$ and $\gamma_{dp}$ by analyzing the trend of the observed $Z_{hh}$ and $Z_{dr}$ with respect to $\Phi_{dp}$ after minimizing their intrinsic variability. Based on the relationship between $\alpha_{hh}$ and $K_{dp}$, Bringi et al. (2001) proposed a self-consistent scheme to optimize $\gamma_{hh}$ by minimizing the discrepancies between observed and reconstructed (through retrieved $\alpha_{hh}$) $\Phi_{dp}$. The latter scheme is adopted here for both PFV and PCA. From scattering simulations based on the T-matrix solution technique (Mishchenko 2000), the microphysical parameterization summarized in Straka et al. (2000), and assuming temperature variations between 5° and 30°C, it can be argued (see Fig. 1) that linearity between $\alpha_{hh}$ and $\alpha_{dp}$ ($\alpha_{dp} = \kappa \alpha_{hh}$) is a relatively fair assumption at C band. Slight deviations from linearity are possible, especially for large raindrop diameters. In Testud et al. (2000), the authors assumed a nonlinear relationship between $\alpha_{dp}$ and $\alpha_{hh}$ with the multiplicative factor being related to the intercept parameter $N_w$ of an assumed gamma drop size distribution. Due to the fact that the estimated $N_w$ is sensitive to absolute calibration, determined through the retrieved profile of $\alpha_{hh}$ and corrected $Z_{hh}$, the suggested approach might be unstable even for a relatively well maintained radar system. Using the idea suggested by Smyth and Illingworth (1998), Bringi et al. (2001) proposed an optimization scheme for $\gamma_{dp}$ based on the physical properties of raindrops in the stratiform tail of the rain cell. Because such regions are not always present in the volume of atmosphere observed by the radar, especially for higher-elevation scans, this approach may not always be usable. To investigate algorithms that are robust enough to be used in an operational context, here we implemented PFV and PCA, assuming a fixed linear relationship between $\alpha_{hh}$ and $\alpha_{dp}$.

4. Adaptive $\Phi_{dp}$ method (APDP) for attenuation correction

In this work the well-known temperature dependency of $\gamma_{hh,dp}$ was taken into account by reconstructing the temperature profile from the observed freezing level height (FLH) retrieved from both radio sounding and aircraft measurements, assuming a standard environmental lapse rate. This approach is prone to uncertainties because of the space–time variability of the freezing level height and temperature lapse rate. In the future, the FLH will be estimated from the operational melting layer identification scheme proposed by Tabary et al. (2006b). Moreover, underestimating (overestimating) the environmental temperature of about 5°C results in an overestimation (underestimation) of $\gamma_{hh}$ of about 10% (Jameson 1992). As mentioned in section 3b, scattering simulations were performed in order to investigate the size dependency of $\gamma_{hh,dp}$. Rain is classified, on the basis of different RSD, into the following four main categories: light rain (LR), moderate rain (MR), heavy rain (HR), and large drops (LD). Table 1 lists the different RSDs adopted in this work. The axis ratio relationship proposed by Brandes et al. (2002) was adopted in this work, because it is a good model for rain commonly observed in northern France (Gourley et al. 2006b). Finally, the dielectric constant was computed as a function of temperature and wavelength according to the model described in Ray (1972). Figure 2 shows the relationship between specific attenuation (differential attenuation) and specific differential phase as obtained from scattering simulations for a restricted range of temperature (9° < T < 11°C). The effect of RSD variability on $\gamma_{hh,dp}$ is significant, and suggests the need for using suitable values for each condition in order to avoid biased estimation of cumulative attenuation. An

<table>
<thead>
<tr>
<th>TABLE 1. Microphysical parameterization used to perform scattering simulations.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Rain class</td>
</tr>
<tr>
<td>LR</td>
</tr>
<tr>
<td>MR</td>
</tr>
<tr>
<td>HR</td>
</tr>
<tr>
<td>LD</td>
</tr>
</tbody>
</table>
adaptive version of the basic $\Phi_{dp}$-based method (APDP) is proposed in this work. As mentioned before, $\gamma_{h,dp}$ is sensitive to both the shape and size distributions of raindrops. A preliminary classification of rain conditions (i.e., LR, MR, HR, and LD) and, consequently, the associated average RSD, would enable the use of suitable values of $\gamma_{h,dp}$ obtained from scattering simulations or disdrometer measurements. For such a purpose, $Z_h$ and $Z_{dr}$, preliminarily corrected using the fixed $\gamma_{h,dp}$, are jointly used with the estimated $K_{dp}$ and $\rho_{hv}$, adopting a Bayesian classification scheme. This approach represents a fully polarimetric version of the algorithm recently proposed by Marzano et al. (2008, hereafter MSMV). As shown in Fig. 3, the maximum a posteriori probability criterion (MAP) is applied to identify the different rain conditions. For a given measurements vector $Y = [Z_h, Z_{dr}, K_{dp}, \rho_{hv}]$, the conditional probability density function (PDF) of a considered rain class $R_c$ can be expressed by means of Bayes’ theorem (Marzano et al. 1999; MSMV; Marzano et al. 2007) as
\[ p(R_c | Y) = \frac{p(Y | R_c)p(R_c)}{p(Y)} = \frac{p(\Delta Y_{R_c})p(R_c)}{p(Y)}, \tag{11} \]

where \( \Delta Y_{R_c} = Y - \langle Y_{R_c} \rangle \) is the perturbation of the measurement vector from the mean value \( \langle Y_{R_c} \rangle \) of class \( R_c \), and \( p(R_c) \) represents the a priori discrete probability of class \( R_c \). The MAP retrieval of a rain class \( R_c \) corresponds to the following maximization with respect to \( R_c \):

\[ \hat{R}_c = \text{Mode}_{R_c}[\ln p(R_c | Y)], \tag{12} \]

where Mode_{R_c} is the modal value of PDF with respect to \( R_c \). Assuming that \( p(\Delta Y_{R_c}) \) is a multivariate Gaussian PDF,

\[ p(\Delta Y_{R_c}) = \frac{1}{\sqrt{(2\pi)^n \det(C_{R_c})}} \exp \left\{ -\frac{1}{2} (Y - \langle Y_{R_c} \rangle)^T C_{R_c}^{-1} (Y - \langle Y_{R_c} \rangle) \right\}, \tag{13} \]

then the most likely rain class is found by minimizing with respect to \( R_c \) the following cost function:

\[ F = -(Y - \langle Y_{R_c} \rangle)^T C_{R_c}^{-1} (Y - \langle Y_{R_c} \rangle) - \ln[\det(C_{R_c})] + 2 \ln[p(R_c)] \tag{14} \]

where \( C_{R_c} \) is the covariance matrix of class \( R_c \). Computing (14) requires knowledge of the measurement mean vector \( \langle Y_{R_c} \rangle \) and covariance matrix \( C_{R_c} \) of each rain class \( R_c \). In this work, the statistical characterization of each rain class has been derived from scattering simulations according to the rain parameterization described in Straka et al. (2000) and MSMV. The prior probability \( p(R_c) \), which was assumed to be uniform, could be used to subjectively weight each class as a function of other available information (i.e., numerical models, disdrometer measurements). The classification is considered to be successful if the minimum of (14) is below a given threshold determined by performing tests on simulations. The Bayesian approach adopted here is substantially different and alternative to the fuzzy-logic methodology, which is based on the probabilistic information. Within the class of model-based algorithms, it requires a much easier setup. Specifically, it is straightforward to derive the new autocovariance matrices once a new microphysical (or scattering) model is specified. The a priori information can be also inserted in a more rigorous form. It has been shown that the performances of the Bayesian classification of hydrometeors are better than, or at least comparable to, the fuzzy-logic approach using two or three polarimetric observables (MSMV). The fuzzy-logic-based method may have the advantage of a much easier empirical tuning through the arbitrary modification of the membership functions (Marzano et al. 2007).

Once the classification is performed at each range gate \( r \), the following \( K_{dp} \)-weighted average of \( \gamma_{hh,dp} \) is used in (10):

\[ \langle \gamma_{hh,dp} \rangle_{K_{dp}} = \frac{\int_{r_0}^{r} K_{dp}(s) \gamma_{hh,dp}^{(R_c)}(s) ds}{\int_{r_0}^{r} K_{dp}(s) ds}, \tag{15} \]

where \( \gamma_{hh,dp}^{(R_c)}(r) \) is the value of \( \gamma_{hh,dp} \) corresponding to the rain class \( R_c \) detected at the range distance \( r \). The reason of this choice is the need to properly weight the values of \( \gamma_{hh,dp} \) according to their contribution to the rain path attenuation.

The APDP method is applied once again using \( Z_{hh} \) and \( Z_{dp} \) corrected in the first step to tune the rain-type classification and, consequently, the final attenuation correction. It is worth mentioning that the proposed classification scheme can also be used for quantitative precipitation estimation by choosing the most suited \( Z - R \) relationship according to the identified rain class.

5. Experimental results

a. Data processing and experimental dataset

The events analyzed in the present work were observed in the Paris area by the C-band Doppler weather radar system operating continuously as part of the
French operational radar network. The radar is equipped with linear polarization capabilities (it transmits horizontally and vertically polarized waves). The two receiving channels, which have nearly identical waveguide runs, operate in parallel, and thus enable the simultaneous transmission and reception (STAR) mode of the polarized signals. According to the adopted scan strategy, data are collected at six different elevation angles every 5 min, which comprises a single cycle. The second cycle begins by repeating the lowest four tilts, but changes the elevation angles at the two highest tilts. The same logic is used for the third and final cycle of the total 15-min volume scan. The antenna diameter is 3.7 m, leading to a beamwidth of about 1.1°, while the pulse width is 2 µs. A staggered pulse-repetition time (PRT; 379, 321, and 305 Hz) scheme is applied for retrieving and dealiasing the Doppler velocities (Tabary et al. 2006a).

A robust data quality check is routinely applied in order to mitigate radar miscalibration, radome interference, and system offsets (Gourley et al. 2006a). Data collected at vertical incidence (one scan each 15 min) are used to estimate the calibration error on Z_{dr} according to the procedure proposed by Gorgucci et al. (1999). The absolute calibration error estimate is performed by means of the self-consistency principle (Gourley et al. 2006b). The radome influence on Z_{dr} and \Phi_{dp} measurements, which is variable with azimuth, was outlined and corrected in Gourley et al. (2006a). To compensate for system noise and Mie scattering effects, an iterative finite impulse response (FIR) filter is applied to the measured differential phase (Hubbert and Bringi 1995) using a moving window of about 3 km.

As shown in Table 2, six events observed during 2005 and 2006 were analyzed in this study. The occurrences (expressed in percentage) of total differential phase shift show that most of them are of moderate intensity in terms of cumulative attenuation. The case of 23 June 2005, instead, is an extreme convective event, which is very unusual in northern France, characterized by path-integrated attenuation up to about 20 dB. The path-integrated differential attenuation (PIDA) was even larger than 10 dB, as demonstrated by the saturated, low (−10 dB) measured values of Z_{dr} (dBZ). To reduce ground clutter contamination and the influence of the melting layer on the evaluation of the attenuation correction methods, the 1.5 antenna elevation scans were considered here.

### b. Evaluation of the proposed methodology: An extreme convective event

In this section we focus on the convective event observed on 23 June 2005. The statistical evaluation of all analyzed events is discussed in section 4c. As an example, a range profile of polarimetric variables and the corresponding rain-type distribution is shown in Fig. 4. For that specific range profile, the LR class is never detected, with Z_{hh} being mostly higher than 30 dBZ. The MR class is typically associated with medium values of Z_{hh} (20 \leq Z_{hh} \leq 40), low K_{dp}, and relatively high values of \rho_{hv}. Heavy rain is characterized by high reflectivity, differential reflectivity, and a specific differential phase but low correlation coefficient. In the considered example, HR is mostly well detected; nevertheless, hail contamination at range distances larger than 50 km is likely, given the very low values of \rho_{hv}. On the other hand, the increase of K_{dp} suggests that hail/wet hail is mixed with rain.

Plan position indicators (PPIs) of Z_{hh} and Z_{dr} (both expressed in decibels), showing the effects of attenuation correction, are illustrated in Figs. 5 and 6, respectively, relative to the event of 1545 UTC 23 June 2005.

As shown in Fig. 6a, the effect of attenuation is clearly visible, resulting in high azimuthal heterogeneity of the Z_{dr} (dB) PPIs. The considered correction algorithms sensibly reduce the effects of attenuation, nevertheless, as mentioned above, there are still wide areas where the likely presence of wet hail prevents full recovery of the Z_{hh} and Z_{dr} fields.

From Figs. 5a and 6a, it can be seen that there are three main azimuthal sectors where attenuation occurs (labeled as S1−S3). In the remaining part of this section we will focus on the analysis of sectors S1 and S2, which are characterized by slightly different performances of

### Table 2. List of analyzed rain events observed by the dual-polarized radar operating in Trappes. The intensity of each event is evaluated in terms of total differential phase shift.

<table>
<thead>
<tr>
<th>Date</th>
<th>Duration</th>
<th>Type of rainfall</th>
<th>0 &lt; \Phi_{dp} &lt; 25 (%)</th>
<th>25 &lt; \Phi_{dp} &lt; 50 (%)</th>
<th>50 &lt; \Phi_{dp} &lt; 100 (%)</th>
<th>100 &lt; \Phi_{dp} &lt; 150 (%)</th>
<th>\Phi_{dp} &gt; 150 (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>24 Mar 2005</td>
<td>3 h</td>
<td>Stratiform</td>
<td>90.12</td>
<td>8.27</td>
<td>1.60</td>
<td>0.0</td>
<td>0.0</td>
</tr>
<tr>
<td>3 Jun 2005</td>
<td>3 h</td>
<td>Mixed</td>
<td>85.03</td>
<td>9.53</td>
<td>4.67</td>
<td>0.69</td>
<td>0.0</td>
</tr>
<tr>
<td>23 Jun 2005</td>
<td>3 h</td>
<td>Convective</td>
<td>57.82</td>
<td>18.65</td>
<td>15.25</td>
<td>5.72</td>
<td>2.56</td>
</tr>
<tr>
<td>3 Jun 2005</td>
<td>3 h</td>
<td>Mixed</td>
<td>85.03</td>
<td>9.53</td>
<td>4.67</td>
<td>0.69</td>
<td>0.0</td>
</tr>
<tr>
<td>23 Sep 2006</td>
<td>5 h</td>
<td>Stratiform</td>
<td>47.77</td>
<td>44.13</td>
<td>6.42</td>
<td>1.54</td>
<td>0.14</td>
</tr>
<tr>
<td>14 Sep 2006</td>
<td>5 h</td>
<td>Mixed</td>
<td>85.03</td>
<td>9.53</td>
<td>4.67</td>
<td>0.69</td>
<td>0.0</td>
</tr>
</tbody>
</table>
the analyzed algorithms. For the sake of brevity, results referring to sector S3 are not explicitly discussed because they are similar to those obtained for sector S2.

1) Analysis of Sector S1

Figure 7 displays plots of the observed and corrected $Z_{hh}$ and $Z_{dr}$ (both in decibels) as a function of the azimuth for a fixed range distance (80 km) in the azimuthal sector comprised between 100° and 120°. Considering that $Z_{dr}$ (dB) is expected to be positive in rain, it is fair to say that APDP reduces differential attenuation slightly more than PFV and PCA. The azimuthal distribution of rain class occurrences along the range (lower panel of Fig. 7) shows that HR and MR are most frequent, with a significant concentration of LD in the final part of the considered sector. Figure 8 shows the
range profile of observed and corrected $Z_{hh}$ and $Z_{dr}$ (both in decibels), $\Phi_{dp}$, and the corresponding identified rain categories relative to the 110.5° azimuth. In spite of the likely hail contamination between 55 and 60 km, APDP is generally able to compensate for the path-integrated attenuation and differential attenuation, which are of the order of 10 and 5 dB, respectively. In this specific case, APDP corrects more than PFV and PCA, with the differences being even larger than 5 dB (1.5 dB).

2) Analysis of sector S2

The impact of the rain–hail mixture on the attenuation correction results appears more significant in the azimuthal sector between 25° and 70°. The large variability of size distribution and water coat thickness makes the attenuation resulting from wet ice very difficult to estimate. A procedure to compensate for attenuation resulting from wet hail has been recently proposed for dual-wavelength systems (Liu et al. 2006). For single-frequency radar that problem is still an open issue. From experience with analysis of the event of 23 June 2005, and on the basis of several hail reports, it is suggested that this problem could be the main cause of uncompensated attenuation. As a matter of fact, hail is commonly observed to be spheroidal in shape. Consequently, the fall mode is crucial in determining the polarimetric signatures of hail. Even if most observations refer to isotropic behavior resulting from tumbling and gyrating motions (Knight and Knight 1970; Bringi et al. 1984), hail can fall with the semimajor axis oriented either in the vertical (Zrnić et al. 1993) or in the horizontal (Smyth et al. 1999). In the latter case, wet hail is expected to produce non-null differential reflectivity (dB) and differential attenuation. Figure 9 shows the

![Fig. 5. PPI of $Z_{hh}$ (dBZ) relative to the event of 1545 UTC 23 Jun 2005. (a) Observed $Z_{hh}$, and $Z_{hh}$ corrected with (b) APDP, (c) PFV, and (d) PCA are shown. The main azimuthal sectors characterized by strong attenuation are labeled in (a) as S1–S3.](image-url)
plots of the observed and corrected $Z_{hh}$ and $Z_{dr}$ as a function of the azimuth for a fixed range distance (50 km). In that sector, the estimated path-integrated attenuation and differential attenuation reach 20 and 10 dB, respectively, with a differential phase shift even larger than 200°. Considering that specific attenuation is typically 3 times larger than specific differential attenuation, the PIA has probably been underestimated in that area. The lower panel of Fig. 9 shows the occurrences (%) of identified rain class along the range as a function of the azimuth. It can be seen that heavy rain is the most frequently identified rain class, while the occurrence of large drops is significant, especially in the first part of the considered azimuthal sector where it ranges between 20% and 35%. It is worth mentioning that a relevant misclassification is likely to happen because of ice contamination whose identification is beyond the scope of the present work. Because negative values of $Z_{dr}$ still persist after correction, we can deduce that none of the considered algorithms is able to fully correct attenuation; moreover, it is noted that PFV compensates for attenuation more than APDP and PCA in the sector between 30° and 45°. On the other hand, APDP performs better than PFV and PCA in the sector between 45° and 65°, as confirmed by the range plot displayed in Fig. 10. The lower panel of Fig. 10 shows the range distribution of identified rain classes. As expected, where the corrected $Z_{dr}$ is still negative the classification algorithm does not provide results, with the minimum of (14) being bigger than the assumed threshold.

c. Evaluation of the proposed methodology: Extensive validation

Quantitative validation of attenuation correction procedures is a cumbersome task. The comparison be-
 tween the radar rainfall estimate and rain gauge measurements is prone to several uncertainties, such as the raindrop size distribution variability, which affects the radar rainfall retrieval by means of a fixed $Z - R$ relationship. In the present work, the physically based approach proposed by Smyth and Illingworth (1998) is adopted to have an independent estimate of the PIDA. Given the linear relationship between $\alpha_{hh}$ and $\alpha_{dp}$ (i.e., $\alpha_{dp} \equiv 0.3 \alpha_{hh}$), it also represents a validation for the estimation of PIA. The aforementioned approach is based on the assumption that, in the stratiform tail of the rain cell (where $Z_{hh} < 20 \text{ dBZ}$), $Z_{dr}$ is expected to be close to zero given the predominant spherical shape of raindrops. As suggested in Bringi et al. (2001), for higher values of $Z_{hh}$, the average intrinsic value of $Z_{int}^{dr}$ can be estimated as a function of intrinsic reflectivity $Z_{int}^{hh}$.

**FIG. 7.** Plot of observed and corrected (a) $Z_{hh}$ and (b) $Z_{dr}$, as a function of the azimuth (between 100° and 120°) for a fixed range distance (80 km) relative to the event of 1545 UTC 23 Jun 2005, and (c) the corresponding $\Phi_{dp}$ and (d) identified rain classes (i.e., LD, LR, MR, and HR).
where $r_N$ denotes the range distance at which the rain cell ends.

The estimate of $Z_{\text{hh}}^{\text{int}}$ requires attenuation compensation; therefore, a basic correction of $Z_{\text{hh}}$ is initially performed by applying (8) using a fixed value of $\gamma_{\text{hh}}$.

Adopting this approach, the PIDA can be estimated as

$$Z_{\text{dr}}^{\text{int}}(r_N) = \begin{cases} 
0 & \text{for } Z_{\text{hh}}^{\text{int}}(r_N) \leq 20 \text{ dBZ} \\
0.048 Z_{\text{hh}}^{\text{int}}(r_N) - 0.774 & \text{for } 20 < Z_{\text{hh}}^{\text{int}}(r_N) \leq 45 \text{ dBZ},
\end{cases}$$

for $Z_{\text{hh}}^{\text{int}}(r_N) \leq 45 \text{ dBZ}$.
the difference between the observed and intrinsic values of $Z_{dr}$, while the error made by the correction procedures can be quantified as

$$
\varepsilon = Z_{dr}^{\text{corr}} - Z_{dr}^{\text{int}}. 
$$

As a matter of fact, 23 June 2005 is an anomalous event for northern France and its analysis should not bias general conclusions that can be drawn for more standard cases. Figure 11 highlights the error analysis performed in terms of mean error $\langle \varepsilon \rangle$ and root-mean-square error relative to all of the examined algorithms (including the standard PDP) for all events listed in Table 2 but the 23 June 2005 case. The estimated errors are plotted as a function of differential phase shift measured at $r_N$ in order to evaluate the performance at increasing cumulative attenuation. It is important to outline that the curve referring to the mean difference
between uncorrected and intrinsic $Z_{dr}$ (Fig. 11a) provides an estimate of mean path-integrated differential attenuation, which reaches almost 3.5 dB. As expected, the PDP method is characterized by a lower standard deviation but higher mean error. Generally, the various methodologies provide comparable performances, with PFV showing the lower RMSE corresponding to larger values of $\Phi_{dp}$, and the maximum difference being about 1.3 and 0.5 dB in terms of mean error and RMSE, respectively. Figure 12 shows the error analysis relative to the event of 23 June 2005, characterized by a mean path-integrated attenuation up to 7.5 dB. In this case, the differences between the various algorithms are more significant, with APDP performing better than the rain profiling algorithms PFV and PCA, which are generally comparable. As expected, PDP is conditioned
by the high space–time variability of the observed RSD. The maximum difference between APDP and PDP is about 2 dB in terms of both mean error and RMSE.

6. Summary

As outlined by several authors, attenuation correction schemes making use of differential phase may be in error because of the temperature, shape, and size distribution dependency of the assumed linear relationship between specific attenuation and specific differential phase. An optimized version of the standard $\Phi_{dp}$-based method for attenuation correction is evaluated in this work. A preliminary classification of the prevailing rain category (drop size distribution) is accomplished by adopting the maximum a posteriori probability criterion by means of the estimated $K_{dp}$, $\rho_{hv}$, and $Z_{hh}$, $Z_{dr}$ corrected using a fixed $\alpha_{ph}-K_{dp}$ relationship. The identified rain types enable the use of appropriate $\alpha_{ph}-K_{dp}$ relationships determined from scattering simulations. A comparison with other advanced polarimetric techniques is performed on six events observed in the Paris area by the dual-polarized radar operating in Trappes. A physically based independent estimate of the path-

**Fig. 11.** Comparison of the considered attenuation correction algorithms (i.e., PDP, APDP, PCA, and PFV) relative to all analyzed events except for 23 Jun 2005. The error analysis is performed in terms of $\langle e \rangle$ and RMSE as a function of differential phase shift.
integrated differential attenuation is used as a reference to evaluate the various correction techniques. Experimental results, based on the moderate events considered, indicate that methods making use of differential phase shift are comparable with rain profiling algorithms in terms of cumulative attenuation estimation, confirming theoretical studies conducted by other authors. As shown by the analysis of an extreme event, the proposed approach better tracks space variability of raindrop size distribution. Expected limitations are encountered in presence of hail/wet ice along the rain path, which still represents an open issue for single-wavelength radar systems. Moreover, the use of differential phase shift for attenuation correction through an adaptive scheme makes the proposed methodology particularly suitable for operational purposes.

Acknowledgments. This work has been supported by the European Community under the FLYSAFE project (Contract AIP4-CT-2005-516167), which is an integrated project of the sixth framework program, consisting of 36 partners from the aviation industry, meteorological services, research centers, and universities.

The author GV is grateful to Dr. D. Scaranari for his fundamental contribution in developing the classification algorithm used in this work.

Fig. 12. Comparison of the considered attenuation correction algorithms (i.e., PDP, APDP, PCA, and PFV) relative to the event of 23 Jun 2005. The error analysis is performed in terms of $\langle e \rangle$ and RMSE as a function of differential phase shift.


APPENDIX

Rain Profiling Algorithms

A brief derivation of the so-called profiling techniques for attenuation correction is given here. Starting from (7), and assuming \( \alpha_{hh} \) is related to reflectivity \( Z_{hh} \) (\( \text{mm}^6 \text{m}^{-3} \)) through a power law, then

\[
\alpha_{hh} = a Z^b. \tag{A1}
\]

Assuming \( b \) is constant in range, (A1) can be rewritten as a differential equation (Iguchi and Meneghini 1994), which takes the form of a Riccati differential equation (Abramowitz and Stegun 1972). A general solution to this differential equation is

\[
Z_{hh}(r) = Z_{hh}^0 \left[ C_1 - S(r) \right]^{-1/b}, \tag{A2}
\]

where \( S(r) = q f_c^r a Z_{hh}(s)^b \, ds \) and \( q = 0.2 \, b \, \ln(10) \). It is worth mentioning that the same approach can be followed to derive the specific differential attenuation \( \alpha_{dd} \) as a function of differential reflectivity \( Z_{dd} \). If \( Z_{hh}(r_0) = Z_{hh}(r_0) \) is taken as a boundary condition, then the HB solution is obtained (Hitschfeld and Bordan 1954) by

\[
Z_{hh}^{\text{HB}}(r) = Z_{hh}^0 \left[ 1 - S(r) \right]^{-1/b}, \tag{A3}
\]

\[
\alpha_{hh}^{\text{HB}}(r) = \frac{a Z_{hh}^0 (r)^b}{1 - S(r)}. \tag{A4}
\]

To avoid the instability of the solution, the boundary condition at the farthest range bin \( (r = r_n) \) can be applied to constrain the solution. Defining the attenuation factor \( A_f \) as

\[
A_f = \exp \left[ -0.46 \int_{r_0}^{r_n} \alpha_{hh}(s) \, ds \right], \tag{A5}
\]

it is possible to derive the so-called FV solution of

\[
Z_{hh}^{\text{FV}}(r) = Z_{hh}^0 \left[ A_f^b + S(r_n) - S(r) \right]^{-1/b}, \tag{A6}
\]

\[
\alpha_{hh}^{\text{FV}}(r) = \frac{a Z_{hh}^0 (r)^b}{A_f^b + S(r_n) - S(r)}. \tag{A7}
\]

It is known that the HB solution can become unstable in heavy rain. The FV solution, on the other hand, is stable using the PIA as a boundary condition. To satisfy both the initial and PIA conditions, the CA method proposed in Meneghini et al. (1983) was modified by Meneghini and Nakamura (1990) to adjust the radar constant introducing the correction factor \( \varepsilon = (1 - A_f^b)/S(r_n) \). The CA solution is given by

\[
Z_{hh}^{\text{CA}}(r) = \varepsilon^{1/b} Z_{hh}^0 \left[ 1 - \varepsilon S(r) \right]^{-1/b}, \tag{A8}
\]

\[
= Z_{hh}^0 \left( \frac{A_f^b}{\varepsilon} - \left[ S(r_n) - S(r) \right] \right)^{-1/b},
\]

\[
\alpha_{hh}^{\text{CA}}(r) = \frac{a(r) Z_{hh}^0 (A_f^{-b} - 1)}{S(r_n) + (A_f^{-b} - 1)[S(r_n) - S(r)]}, \tag{A9}
\]

which, after a few mathematical manipulations, and assuming \( a \) is constant in range, takes the form

\[
\alpha_{hh}^{\text{CA}}(r) = \frac{(Z_{hh}^0 (A_f^{-b} - 1))}{I(r_0 r_n) + (A_f^{-b} - 1)I(r r_n)}, \tag{A10}
\]

where

\[
I(r r_n) = q \int_r^{r_n} Z_{hh}^0 (s) \, ds.
\]

The above profiling techniques (FV and CA), also called surface reference methods, estimate the PIA through rain from the decrease in surface return; in particular, the loss factor \( A_f \) at \( r = r_n \) is estimated as the ratio between the surface return power in rain to that measured in adjacent rain-free areas.

REFERENCES


