Model-Based Prediction of Amplitude Scintillation Variance Due to Clear-Air Tropospheric Turbulence on Earth-Satellite Microwave Links

Frank S. Marzano and Giovanni d’Auria

Abstract—A statistical method to predict tropospheric amplitude scintillation parameters along earth-space microwave links from meteorological data is proposed. The evaluation of the mean value and the variance of the refractive-index structure constant and of the scintillation power (i.e., the variance of the log-amplitude fluctuations of the received electromagnetic field) is carried out from conventional radio-sounding measurements. A large radio-sounding data set, collected in Northern Italy over ten years is utilized to simulate clear-air amplitude scintillation variance at microwaves and millimeter-waves on slant paths. Scintillation statistics of interest for link-budget design are also derived from the radio-sounding data set for short and long-term applications. Scintillation prediction formulas, based on measurements of surface temperature and relative humidity, are also derived and regression coefficient tables are given on an hourly and a monthly basis. Comparisons of short-term and long-term prediction results with Olympus down-link measurements at 19.8 GHz are shown and discussed. A model investigation about the statistical correlation between scintillation power and brightness temperature is performed, deriving an extension of the estimation methods to include integrated water vapor measurements from ground-based microwave radiometers.

Index Terms— Meteorology, microwave propagation, random media, satellite communication.

I. INTRODUCTION

One of the major problems in the link budget design of microwave and millimeter-wave communication systems is represented by tropospheric scintillation, i.e., rapid fluctuations of the signal amplitude and phase due to tropospheric turbulence [1]. Scintillation phenomena can cause a significant degradation of the signal-to-noise ratio (up to several decibels) and their effects increase with the increase of channel frequency and the decrease of the antenna aperture and elevation angle [2], [3]. The analysis of the tropospheric scintillation impact is gaining a renewed interest due to the introduction of mobile and fixed digital telecommunications and elevation angle [2], [3]. The analysis of the tropospheric scintillation impact is gaining a renewed interest due to the introduction of mobile and fixed digital telecommunications and elevation angle [2], [3]. The analysis of the tropospheric scintillation impact is gaining a renewed interest due to the introduction of mobile and fixed digital telecommunications and elevation angle [2], [3]. The analysis of the tropospheric scintillation impact is gaining a renewed interest due to the introduction of mobile and fixed digital telecommunications and elevation angle [2], [3]. The analysis of the tropospheric scintillation impact is gaining a renewed interest due to the introduction of mobile and fixed digital telecommunications and elevation angle [2], [3]. The analysis of the tropospheric scintillation impact is gaining a renewed interest due to the introduction of mobile and fixed digital telecommunications and elevation angle [2], [3]. The analysis of the tropospheric scintillation impact is gaining a renewed interest due to the introduction of mobile and fixed digital telecommunications and elevation angle [2], [3]. The analysis of the tropospheric scintillation impact is gaining a renewed interest due to the introduction of mobile and fixed digital telecommunications and elevation angle [2], [3]. The analysis of the tropospheric scintillation impact is gaining a renewed interest due to the introduction of mobile and fixed digital telecommunications and elevation angle [2], [3]. The analysis of the tropospheric scintillation impact is gaining a renewed interest due to the introduction of mobile and fixed digital telecommunications and elevation angle [2], [3]. The analysis of the tropospheric scintillation impact is gaining a renewed interest due to the introduction of mobile and fixed digital telecommunications and elevation angle [2], [3].

The tropospheric turbulence is represented by tropospheric scintillation, i.e., rapid fluctuations of the received electromagnetic field

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in order to introduce the atmospheric parameters estimated by microwave radiometers within model-based scintillation prediction methods.

In this work, a model of $C_n^2$, which takes into account the turbulence intermittence as a random process, is applied. The formulation of the random model of $C_n^2$ basically involves both the Richardson number to describe the local atmospheric instability and the Tatarskii theory of homogeneous turbulence. In Section II, the calculation of the mean value and variance of $C_n^2$ is carried out by expressing the mean value of $C_n^2$ in intermittent turbulence through mean values of meteorological variables and gradients, directly derivable from conventional radio-sounding observations (RAOB’s). In Section III, a ten-year RAOB data set is used in order to simulate the statistical distribution of the structure constant and of the received scintillation power for a microwave slant link at 19.8 GHz and elevation angle of 30.6°. In Section IV, scintillation prediction formulas from surface meteorological data are developed using the multivariate regression analysis. Comparisons of short-term and long-term prediction results with Olympus downlink measurements at 19.8 GHz are shown and discussed. Finally, in Section V, a model investigation of the statistical relationship between scintillation variance and microwave brightness temperature is carried out.

II. MODELING SCINTILLATION IN INTERMITTENT TURBULENCE

Among the approaches based on meteorological measurements, the model proposed by Tatarskii has been the most used for calculating the refractive-index structure constant $C_n^2$ in clear air [19]. The Tatarskii model of the microstructure of the refractive index in turbulent flow relates the value of the refractive-index structure constant $C_n^2$ to the outer scale of turbulence $L_o$ and to the vertical gradient $M$ of refractivity, assuming a statistically stationary regime and a well-developed homogeneous turbulence following the Kolmogorov law [19], [20]. Actually, in the real atmosphere this condition is only seldom met because the clear air may be locally fluctuating between unstable and stable conditions [21]. Measurements of clear-air turbulence have shown that turbulence may be found in thin layers with sharp randomly varying boundaries, strongly related to the wind shear instability, i.e., turbulence exhibits intermittence phenomena [22], [23].

The intermittent nature of turbulence leads to the need for a local stability criterion, which can be introduced by means of the Richardson number $R_i$ defined as $R_i = B/S$ where $B$ is the atmospheric buoyancy and $S$ is the square wind shear. Only where $R_i$ is less than or equal to the critical Richardson number $R_{ic}$ (equal to 0.25), the stratification is locally unstable and turbulence is developed so that the Tatarskii model could be directly applied. It has been shown that $C_n^2$ is mainly related to the structure constants of temperature and humidity, i.e., to their local vertical gradients, and the transverse wind shears [24]. In order to define a random model of $C_n^2$, the refractive index structure constant may be supposed as a strongly nonlinear function of the outer scale $L_o$, of the square wind shear $S$, and of the vertical gradient $M$ of refractivity (i.e., explicitly of the buoyancy $B$ and of the humidity gradient $Q$) and can be expressed by [22]

$$C_n^2(L_o, S, M) = a^2L_o^{\alpha/3}M^2u(S - S_c)$$

(1)

where $a^2 = 2S$ and $u(S - S_c)$ is the step function centered on the “critical” square shear $S_c$ defined as: $S_c = B/R_{ic}$. The expression of the refractivity vertical gradient is $M = (\xi B + \zeta Q)^2$, being $\xi$ and $\zeta$ functions of pressure, temperature and humidity and $Q$ the vertical gradient of specific humidity. Equation (1) describes the intermittence effects on $C_n^2$, which is statistically characterized by its probability density function $p(C_n^2)$.

A. Mean Value and Variance of Refractive-Index Structure Constant

The substantial lack of small-scale measurements of the statistical distribution of $C_n^2$ leads one to perform the calculation of the mean value of $C_n^2$ in four-dimensional space ($L_o$, $S$, $B$, $Q$). The calculation may be simplified assuming: 1) as first approximation, the refractivity gradient $M$, the outer scale $L_o$, and the square wind shear $S$ to be statistically independent of each other; 2) since the relative fluctuations of $B$ about its large-scale mean value are often small with respect to those of $S$ in free atmosphere, the value of $S_c$ to be a constant given by $S_c = (B)/R_{ic}$; and 3) the correlation coefficient between $B$ and $Q$ to be equal to ±1.

Using the above hypotheses and noting that generally $\langle M \rangle^2 \gg \sigma_M^2$, the mean value of $C_n^2$ (in $m^{-2/3}$) can be expressed as [22]

$$\langle C_n^2 \rangle \approx a^2L_o^{\alpha/3}F_S\langle M \rangle^2 = a^2L_o^{\alpha/3}F_S\langle \xi B + \zeta Q \rangle^2$$

(2)

where the effective outer scale $L_{oe}$ and the intermittence factor $F_S$ are defined as

$$F_S = \int_{S_c}^{\infty} p(S) dS$$

(2b)

where $L_{oe}$ and $L_{om}$ are, respectively, the minimum and the maximum value experimentally found for $L_o$.

The scarce availability of small-scale observations still gives rise to difficulties in choosing $p(L_o)$ and $p(S)$. Several probability density functions (pdf’s) may be assumed for the shear or for the square shear $S$. If the horizontal components of the wind-shear vector are supposed normally distributed with the same standard deviation, the shear results distributed according to the Rice–Nagakami pdf. The factor $L_{oe}$ may be evaluated assuming, for instance, a uniform value for $p(L_o)$ between $L_{om}$ and $L_{om}$. It is worth noting that if there is no intermittence and the turbulence is well-developed and homogeneous, then $p(S) = 0$ for $S < S_c$ and $L_o$ is a constant. In this case, as expected, the factor $F_S$ reduces to one and (2) becomes equal to the Tatarskii expression.

Following the same approach as described above, it is also possible to calculate the variance of the refractive-index structure constant $\sigma_{C_n^2}^2$, i.e.,

$$\sigma_{C_n^2}^2 = \int_0^{\infty} (C_n^2 - \langle C_n^2 \rangle)^2 p(C_n^2) dC_n^2 = \langle (C_n^2)^2 \rangle - \langle C_n^2 \rangle^2$$

(3)
Substituting the expression of $C_n^2$ given in (1) into (3), we can express the term $\langle (C_n^2)^2 \rangle$ of the previous relationship as
\[
\langle (C_n^2)^2 \rangle = \int_{-\infty}^{+\infty} \int_{L_{0m}}^{L_{0m}} \int_{S_0}^{\infty} (d^2 L_{0m}^{\delta/3} M^2)^2 \rho(L_0, S, M) \cdot dS \, dL_0 \, dM.
\] 
(4)

Under the same assumptions valid for obtaining (2), results that
\[
\langle (C_n^2)^2 \rangle = a_1^4 F_S(\langle M \rangle^4 + 2\sigma_{M^2}^2 \langle M \rangle^2 + \sigma_{M^2}^2) \cdot \int_{L_{0m}}^{L_{0m}} L_0^{\delta/3} \rho(L_0) \, dL_0 \cong a_1^4 F_S(\langle M \rangle^4 L_{0w}^{\delta/3})
\] 
(5)

where the last approximation is allowed considering that $\sigma_{M^2}^2 \ll \langle M \rangle^4$ and $\sigma_{M^2}^2 \ll \langle M \rangle^4$ and we have defined
\[
L_{0w}^{\delta/3} = \int_{L_{0m}}^{L_{0m}} L_0^{\delta/3} \rho(L_0) \, dL_0.
\] 
(6)

Then, substituting (2) and (5) into (3), we obtain the following approximated expression:
\[
\sigma_{C_n}^2 \cong a_1^4 F_S L_{0w}^{\delta/3} - F_S L_{0w}^{\delta/3} (\langle M \rangle^4).
\] 
(7)

The last equation shows that the variance of $C_n^2$ can be locally evaluated by knowing the meteorological quantities needed for the estimation of the mean value of $C_n^2$. It is interesting to analyze the implications of (7). If $L_o$ is supposed to be constant and equal to $L_{0w}$ (i.e., $\rho(L_0) = \delta(L_o - L_{0w})$ with $\delta$ the Dirac function), then (7) can be expressed in terms of structure-constant standard deviation $\sigma_{C_n}^2$ as
\[
\sigma_{C_n}^2 \cong \alpha_1^4 F_S (1 - F_S) L_{0w}^{\delta/3} (\langle M \rangle^2)^2
\] 
(8)

which can be also rewritten by using (2) as
\[
\sigma_{C_n}^2 \cong \sqrt{(1 - F_S)} F_S \langle C_n^2 \rangle.
\] 
(9)

As physically reasonable, the standard deviation of $\sigma_{C_n}^2$ tends to zero as intermittent turbulence tends to vanish (i.e., $F_S = 0$) or turbulence is well developed and homogeneous (i.e., $F_S = 1$). It is worth noting that $\sigma_{C_n}^2$ is less than $\langle C_n^2 \rangle$ for $F_S > 0.5$, while the opposite holds for $F_S < 0.5$ when $\langle C_n^2 \rangle$ becomes very small.

### B. Evaluation of Log-Amplitude Fluctuation Variance on Slant Paths

The calculation of the amplitude scintillation variance $\sigma_\chi^2$, i.e., the variance of the log-amplitude fluctuations $\chi$ of the received electromagnetic (EM) field, can be carried out by considering $\sigma_\chi^2$ as a random function of the structure constant given in (2).

If the Taylor “frozen-in” hypothesis assumed and the atmospheric turbulence lies in the inertial subrange of Kolmogorov’s spectrum and is intermittent, we can derive the following expression for the mean variance $\langle \sigma_\chi^2 \rangle$ of log-amplitude fluctuations (expressed in decibels) [20]:
\[
\langle \sigma_\chi^2 \rangle = 42.9 k_0^{7/6} \int_0^L \rho(n(r)) r^{5/6} \, dr
\] 
(10)

where $L$ is the path length of the plane wave EM radiation through turbulence, $k_0$ is the wave number in vacuum, and $r$ is the path coordinate. Within a horizontally stratified atmosphere and using the expression given in (2), the evaluation of $\langle \sigma_\chi^2 \rangle$, received by a finite size antenna, can be performed by using the following numerical form of (10) [14]
\[
\langle \sigma_\chi^2 \rangle = 23.2 G^2 \frac{\sigma_{K_0}^{7/6}}{(\sin \theta)^3} \sum_{i=1}^m H_\theta^i H_\theta^{-i/11/6}
\] 
(11)

where $H_\theta$ and $h_\theta$ are, respectively, the height of the upper and lower extreme of the $i$th layer of $m$ turbulent layers, within each $\langle C_n^2(i) \rangle$ is constant, $\theta$ is the elevation angle above the surface, and $G$ is the antenna aperture averaging factor (e.g., see [10] and [11]). The mean temporal-power spectrum $\langle \omega_\chi \rangle$ of log-amplitude fluctuations has a behavior similar to the spectrum in the homogeneous case (characterized by a high-frequency asymptote with logarithmic slope of $-11/3$), but with an area proportional to the mean variance $\langle \sigma_\chi^2 \rangle$.

It is worth mentioning that the use of (10) and (11) can be questionable when dealing with very weak turbulence since in these conditions the inertial subrange of the Kolmogorov spectrum tends to disappear and the spectrum itself might be no more valid. Even generalized scintillation spectra, like those proposed in literature [13], are not applicable because valid only in the case of well-developed turbulence. Since this work was mainly devoted to radiopropagation, the modification of the basic theory of turbulence was beyond our scopes. However, we have tried to parameterized the theory itself in case of very weak turbulence by introducing the intermittence factor $F_S$ [given by (2b)] which modulates in a continuous way the amplitude of scintillation variance, as shown by (11). Ranging from zero to one, the intermittence factor connects the case of laminar flow to that of well-developed turbulence giving an approximation which seems physically reasonable from an EM propagation point of view.

The estimation of $\langle \sigma_\chi^2 \rangle$ can be performed by supposing only one turbulent layer, instead of the multilayer structure assumed in (11). Two simple models can be derived: a slab model where the turbulence layer goes from the surface up to an altitude $H$ and a thin-layer model where the turbulence layer of thickness $d$ goes from an altitude $(H - d)$ up to $H$ being $d \ll H$. From (11) we have for the slab model [20], [25]
\[
\langle \sigma_\chi^2 \rangle = 42.3 G^2 \langle C_n^2 \rangle k_0^{7/6} H^{11/6} \sin \theta^{11/6}
\] 
(12a)

while for the thin-layer model [25]:
\[
\langle \sigma_\chi^2 \rangle = 42.3 G^2 \langle C_n^2 \rangle k_0^{7/6} d^{1/3} H^{11/6} \sin \theta^{11/6}.
\] 
(12b)

The above expressions given in (11), (12a), and (12b) will be compared in the next sections.
III. SIMULATING SCINTILLATION FROM RADIO-SOUNDING DATA

The applicability of the expressions given in (2) and (7) requires at least the knowledge of the variability range of $L_0$, the mean values and the standard deviations of $S$, $B$, and $Q$ from small-scale meteorological data sets. An estimate of $\langle C_n^2 \rangle$ is possible if we consider an atmospheric layer defined by two RAOB measurements at two adjacent altitudes and approximate the mean values of meteorological parameters through their spatial averages within the slab. Furthermore, the fluctuations of $M$ and $S$ about their spatial averages are generally small and may be expressed in terms of the spatial averages themselves [22]. Thus, for a specific RAOB we can carry out an estimation of $\langle C_n^2 \rangle$ evaluating the mean values of meteorological variables and their gradients through the spatial averages of RAOB data. In this context the $p(C_n^2)$ assumes the meaning of probability density of occurrence of a given turbulence within the considered slab and $\langle C_n^2 \rangle$ is intended as a slab spatial-average.

We have examined a data set of RAOB’s with a vertical spatial resolution of about 250 m or smaller, performed in Milan, Italy by Service Meteorologico Aeronautica Militare Italiana between January 1980 and December 1989. Two radio soundings per day at midday and at midnight have been available for a total of 3655 RAOB’s (900, 1025, 840, and 890 samples for spring, summer, autumn, and winter, respectively, while 1800 and 1865 samples for midday and midnight, respectively). An accurate selection of the clear-sky RAOB’s has been carried out in order to exclude cloudy and rainy conditions. Linear interpolation procedures have been applied in order to overcome the lack of data at certain levels. The data set has been classified with respect to the hour (midday or midnight), month and season of the balloon launch.

Any result obtained by using the above RAOB data set will be biased by the location and period of acquisition of the data set itself. In particular, the applicability of the proposed prediction methods from meteorological data should be restricted to climates of subcontinental mid-latitude type.

A. Comparison of Scintillation Models

The contribution of each layer to the profile of $\langle C_n^2 \rangle$ and to the total received power $\langle \sigma^2 \rangle$ can be examined by plotting against the altitude each term of the sum in (11). This analysis is instructive since it gives the physical basis of the statistical results shown in the next sections.

The top-left panel of Fig. 1 shows the vertical profile of the measured temperature $T$ and Richardson number $R_i$ for the RAOB of August 6, 1989 at midday (summer case). Two very stable layers are apparent close to the surface and between 1500 and 3000 m (where $R_i \gg 0.25$), while a well-developed turbulent region is clearly shown between 200 and 1500 m (where $R_i < 0.25$). For altitudes greater than 3000-m turbulence is present with intermediate strength; note that around 3000-m temperature exhibits a thermal inversion corresponding to a high value of $R_i$. The top-right panel shows the profile of the intermittence factor $F_S$ as derived from (2b): it is close to zero in correspondence with the very stable layers close to the surface and between 1500–3000 m, while close to one within the strong turbulent region between 200–1500 m.

The bottom-left panel of Fig. 1 shows the profiles of the mean and standard deviation of $C_n^2$, as calculated from (2) and (11). The mean value $\langle C_n^2 \rangle$ has a minimum around 3200 m where the thermal inversion takes place, while the standard deviation $\sigma_{C_n^2}$ is always slightly greater than $\langle C_n^2 \rangle$ except for the stable regions where $F_S > Q_3$ [see (9)]. Finally, the bottom-right panel shows the vertical profile of the scintillation variance contributions $\langle \sigma^2(i) \rangle$ of the $i$th layer, as computed from (11) and indicated by a solid line (intermittent-turbulence model). The area under the $\langle \sigma^2(i) \rangle$ profile represents the mean received scintillation power $\langle \sigma^2 \rangle$, which is equal to $1.307 \times 10^{-2}$ dB$^2$ in this case. For comparison the dashed line indicates the scintillation variance profile computed by using the formula for $\langle C_n^2 \rangle$ with $F_S = 1$ in (2) (homogeneous-turbulence model). The mean variance of the homogeneous-turbulence model is equal to $\langle \sigma^2 \rangle = 9.923 \times 10^{-2}$ dB$^2$, which is higher than the intermittent-turbulence model. This is simply explained by noting that the intermittence factor is much less than one within the two mentioned stable layers.

A very debated issue in literature is the choice of the top level of the turbulent slab or layer [25], [26]. Since it has been shown that the state of turbulence is mainly correlated to the humidity profile [9], we have chosen to truncate the vertical profiles at the height where the water vapor content was less than 0.001 g/m$^3$. This has led to the introduction of a height $H_t$, which indicates the maximum height below which the atmosphere may be either unstable or stable. In the intermittent-turbulence model, this means that the effective turbulent atmosphere will be equal or less than $H_t$ depending on intermittence. Thus, a rough estimation of the turbulence effective height $H_{te}$ can be given by weighting $H_t$ with the
Fig. 2. Analysis of turbulence layer thickness derived from the ten-year RAOB data set. Turbulence top-height $H_t$ as function of surface temperature (top-left). Histogram of vertically-average intermittence factor $F_S$ (bottom-left panel). Histogram of turbulence effective-height $H_{te}$ (bottom-right panel).

Vertically averaged value $F_{Shva}$ of the intermittence factor, i.e.,

$$H_{te} = H_t F_{Shva}. \quad (13)$$

Fig. 2 shows the analysis of turbulence layer thickness derived from the ten-year RAOB data set. The right- and left-top panels show the correlation of $H_t$ with the surface temperature $T_S$ and the histogram of $H_t$ over ten years, respectively. The bottom-left panel shows the histogram of $F_{Shva}$, while on the right, the histogram of $H_{te}$ as derived from (13). All the figures refer to data on an hourly basis, without distinguishing season and launch time. A large dispersion of data is noted in the scatterplot, even though the correlation of $H_t$ with $T_S$ is greater than 0.55. The mean value of $H$ is 3380 m and its standard deviation of 1332 m, while the mean value of $F_{Shva}$ is 0.43 with standard deviation of 0.23. The range of $H_{te}$ values, characterized by a mean and standard deviation of 1422 and 845 m, respectively, is in a fairly good agreement with the results shown in [25] and [26] and also in [10] (where they suggest a path length of 2000 m).

Using a given RAOB, we have already showed in Fig. 1 the differences between the intermittent and homogeneous turbulence models and their impact on $\langle \sigma_2^2 \rangle$ evaluation. The top panel of Fig. 3 illustrates the scatterplot of the scintillation variances derived from the homogeneous and intermittent turbulence models using the whole ten-year data set. Note that the mean value of the difference between the intermittent and homogeneous turbulence model results is $-0.0954 \text{ dB}^2$, while its standard deviation is $0.0837 \text{ dB}^2$. The fact that $F_S$ is generally less than one explains the lower values of intermittent-turbulence $\langle \sigma_2^2 \rangle$ with respect to the homogeneous turbulence results.

The bottom of Fig. 3 shows the cumulative distribution function (CDF) of $\langle \sigma_2^2 \rangle$ obtained from the whole data set by applying the intermittent-turbulence model [from (11)], the homogeneous-turbulence model [from (11) with $F_S = 1$], the slab-turbulence model [from (12a)] and the thin-layer turbulence model [from (12b)]. For the slab- and thin-layer models we have used the vertical average of $\langle \sigma_2^2 \rangle$ profile derived from RAOB’s, thus also including the intermittence effects; the slab-top height has been made equal to $H_{te}$ (see Fig. 2), while $d = 400$ m is the chosen thickness of the thin layer. Under these assumptions, both the slab- and thin-layer models tend to overestimate the homogeneous turbulence model results, whose values, in turn, are higher than those of the intermittent turbulence model. In particular, the slab-model results are higher than those of the thin layer, as guessed.

B. Statistical Distribution of Scintillation Parameters

The availability of the RAOB data set can be exploited to infer the statistical distribution of scintillation parameters, like $\langle \sigma_2^2 \rangle$ and $\langle C_2^2 \rangle$. In order to resume the main statistical features, we have performed the vertical average of $\langle C_2^2 \rangle$ for each RAOB so that in the following we will refer to its vertically averaged value, indicated by $\langle C_2^2 \rangle_{vt}$. We will also show simulation results on an hourly basis, that are taking each available RAOB during the day, grouped with respect to hour, month, and season. No particular trends have been noticed by analyzing the interannual variability. Considering the ascending time of a radio-sounding balloon, the evaluation of $\langle C_2^2 \rangle$ could correspond to the temporal averaging of about 10-min intervals, a period which is comparable to those experimentally used for long-term analysis [25]. Strictly speaking, we have only two samples (midday and midnight) to make our statistics on an hourly basis: the temporal sampling of radio-sounding launches represents a limit of this approach. However, it is important to stress that over the last years the national meteorological agencies have started to perform routinely four RAOB’s per day, apart from several scientific campaigns performing radio-soundings.

Fig. 4 shows the histograms of $\langle C_2^2 \rangle_{vt}$ and $\langle \sigma_2^2 \rangle$ expressed in logarithmic scale, grouping the ten-year RAOB data set with
With respect to month; only the months of January (297 samples) and August (344 samples) are shown. The normal pdf is also shown on each graph by a solid line; a normalization procedure has been necessary to make the comparison on the same plot by imposing the pdf area to be unitary. The normal pdf has resulted to be the best-fitting pdf among the considered ones (in particular, the \(\Gamma\) distribution, the Rayleigh distribution, and the Rice distribution) after performing a \(\chi\)-square statistical test. Indeed, all the simulated histograms show no noticeable skewness so that it is fairly reasonable to adopt a symmetric distribution on a logarithmic scale. This means that both \(\langle C_n^2 \rangle_{10a}\) and \(\langle \sigma^2 \rangle\) distribution are well approximated by a log-normal pdf on a linear scale (dB\(^2\) and m\(^{-2/3}\), respectively).

In January, the mean value (standard deviation) of \(\langle C_n^2 \rangle_{10a}\) and \(\langle \sigma^2 \rangle\) is \(2.02 \times 10^{-13}\) m\(^{-2/3}\) (1.17 \(\times 10^{-12}\) m\(^{-2/3}\)) and 5.97 \(\times 10^{-3}\) dB\(^2\) (1.64 \(\times 10^{-2}\) dB\(^2\)), respectively; in August it is 6.24 \(\times 10^{-13}\) m\(^{-2/3}\) (3.09 \(\times 10^{-12}\) m\(^{-2/3}\)) and 4.02 \(\times 10^{-2}\) dB\(^2\) (9.58 \(\times 10^{-2}\) dB\(^2\)), respectively. As expected, the mean values and standard deviations in January are always less than those in August (by up to one order of magnitude). Note that the mean values are generally less at midnight than at midday, while the opposite happens for the standard deviations.

Fig. 5 shows the same as in Fig. 4, but grouping the ten-year RAOB data set with respect to season; only winter (890 samples) and summer (1025 samples) seasons are shown. It is worth noting the fairly good fitting of summer’s histogram of \(\langle \sigma^2 \rangle\) by normal pdf as an effect of restricting the grouping criterion, that is selecting more homogeneously the RAOB data set.

In winter the mean value (standard deviation) of \(\langle C_n^2 \rangle_{10a}\) and \(\langle \sigma^2 \rangle\) is 1.68 \(\times 10^{-13}\) m\(^{-2/3}\) (8.88 \(\times 10^{-13}\) m\(^{-2/3}\)) and 7.20 \(\times 10^{-3}\) dB\(^2\) (2.05 \(\times 10^{-2}\) dB\(^2\)), respectively; in summer it is 5.08 \(\times 10^{-13}\) m\(^{-2/3}\) (3.34 \(\times 10^{-12}\) m\(^{-2/3}\)) and 3.40 \(\times 10^{-2}\) dB\(^2\) (7.45 dB\(^2\)), respectively. The mean values and standard deviations in winter are always less than those in summer; note that summer \(\langle C_n^2 \rangle_{10a}\) and \(\langle \sigma^2 \rangle\) values are in a fairly good agreement with those published in [13].

From the analysis of Figs. 4 and 5 we can conclude that the best-fitting long-term pdf of \(\langle \sigma^2 \rangle\) and also \(\langle C_n^2 \rangle_{10a}\) is log-normal on month, season, and year time periods analyzed on an hourly basis. This conclusion is agreement with several experimental results, performed along satellite links with different angles and frequency bands [6], [12], [25]. Thus, we can model \(\langle \sigma^2 \rangle\) pdf as follows:

\[
p(\langle \sigma^2 \rangle) = \frac{1}{\sqrt{2\pi s^2(\langle \sigma^2 \rangle)}} \exp \left[-\frac{(\ln(\langle \sigma^2 \rangle) - m)^2}{2s^2}\right] \tag{14}\]

where \(m\) and \(s\) are the mean and the standard deviation of \(\ln(\langle \sigma^2 \rangle)\), respectively. Hereafter we will refer to \(\ln(\langle \sigma^2 \rangle)\) as scintillation log-variance, expressed conventionally in \(NP\) (when \(\langle \sigma^2 \rangle\) is in dB\(^2\)). The knowledge of the two statistical moments \(m\) and \(s\) completely characterized the \(\langle \sigma^2 \rangle\) distribution and the goal of the next section will be to relate them statistically to meteorological parameters. Moreover, (14) allows us to derive the pdf of mean log-amplitude fluctuations \(\langle \chi \rangle\) on long-term periods as a marginal pdf of the joined pdf \(p(\langle \chi \rangle, \langle \sigma^2 \rangle)\), i.e., [10]

\[
p(\langle \chi \rangle) = \int_0^\infty p(\langle \chi \rangle, \langle \sigma^2 \rangle) d\langle \sigma^2 \rangle
= \int_0^\infty p(\langle \chi \rangle|\langle \sigma^2 \rangle) p(\langle \sigma^2 \rangle) d\langle \sigma^2 \rangle \tag{15}\]

where \(p(\langle \chi \rangle|\langle \sigma^2 \rangle)\) is the short-term conditional pdf of \(\langle \chi \rangle\), introduced through the Bayes theorem. Notice that in (14) and (15) the angle-brackets refer to the ensemble average of small-scale intermittent turbulence phenomena.

There is a substantial consensus in literature to assume \(p(\langle \chi \rangle|\langle \sigma^2 \rangle)\) as Gaussian under statistically stationary conditions on short-term periods that is on what we have called an hourly basis (order of tens of minutes or less). Even though
the scintillation variance can be treated as constant for a short term, it does vary with meteorological conditions. Thus, it is of much interest to infer \( \langle \sigma_n^2 \rangle \) from meteorological data not only on a monthly basis, but also on an hourly basis; this will be another aim of Section IV.

IV. PREDICTING SCINTILLATION FROM SURFACE METEOROLOGICAL DATA

Most of scintillation measurements at ground receiving stations are performed together with surface meteorological measurements such as temperature, relative humidity, and wind velocity. The facility of doing radio soundings close to the satellite ground station is not always planned and more often it is necessary to refer to the closest official RAOB station, which can be many hundreds of kilometers far apart.

Using the statistical multivariate regression method, we have developed several prediction models of amplitude scintillation variance based on the combination of surface temperature and relative humidity. In order to establish the estimation accuracy, we have performed both simulated and experimental tests. As a simulated “truth,” we have used the scintillation variances obtained from the corresponding RAOB profiles. The experimental validation of the prediction models has been carried out by using six months of Olympus satellite measurements, sampled around midday and midnight.

In order to make the estimation method independent of antenna-aperture averaging, frequency, and elevation angle, and having in mind the expression (12a), we have introduced a normalized mean scintillation variance as follows:

\[
\langle \sigma_n^2 \rangle = \frac{\langle \sigma^2 \rangle}{G^2 f^{\alpha}(\sin \theta)^{-\beta}}
\]  

(16)

where \( \alpha \) and \( \beta \) are the frequency scaling exponent and the elevation scaling exponent, respectively. Following the Tatarskii theory, results that \( \alpha = 7/6 \approx 1.16 \) and \( \beta = 11/6 \approx 1.83 \) [see (12a)], while in [10] \( \alpha = 0.00 \) and \( \beta = 2.60 \), in [11] \( \alpha = 1.16 \) and \( \beta = 2.40 \), and in [12] \( \alpha = 1.21 \) and \( \beta = 2.40 \). Consistently with the theoretical framework of (10), within our simulation we have chosen the Tatarskii values for \( \alpha \) and \( \beta \). Note that the introduction of \( \langle \sigma_n^2 \rangle \) follows the same approach presented in [10] and [12]. The antenna averaging factor \( G \) has been assumed to be that suggested by ITU-R [11].

Referring to log-normal pdf of the scintillation log-variance given in (14), the normalization given in (16) implies that

\[
m_{\sigma_n} = \ln\langle \sigma_n^2 \rangle = m_{\sigma} + \ln \left( \frac{1}{G^2 f^{\alpha}(\sin \theta)^{-\beta}} \right)
\]  

(17)

where \( m_{\sigma_n} \) is the mean of normalized scintillation log-variance and the overbar stands for long-term temporal average. It is easy to show that for the standard deviation \( s_{\sigma_n} \) of normalized scintillation log-variance holds \( s_{\sigma_n} = s_{\sigma} \).

A. Scintillation Variance and Surface Meteorological Data

The long-term correlation between the scintillation variance and surface meteorological parameters has been widely exploited, mostly using experimental data [2], [10], [12]. In this section, we will illustrate the results obtained from our simulation in view of developing a prediction model based on ground-based measurements only.

Fig. 6 shows the normalized log-variance on an hourly basis [i.e., \( \ln(\langle \sigma_n^2 \rangle) \)] on left panels and on a monthly basis [i.e., \( m_{\sigma_n} \) from (17)] on right panels against the surface temperature \( T_S \) (top panels) and surface relative humidity \( RH_S \) (bottom panels) obtained from the ten-year RAOB data set. We show the results on an hourly basis in terms of \( \langle \sigma_n^2 \rangle \) and not \( \langle \sigma^2 \rangle \) since the correlation with surface data is generally much higher. A monthly-basis average means that the simulated scintillation parameters associated to each RAOB and surface data have been temporally averaged over a month.

The (linear) correlation between log-variance and \( T_S \) goes from 0.51 on an hourly basis to 0.91 on a monthly basis, while the correlation between log-variance and \( RH_S \) goes from -0.02 to -0.43. Thus, the temporal averaging strongly increases the (linear) correlation between log-variance and each surface variable. This effect it is not related to the specific data set considered in this work, but is relative to a more general conclusion obtained from the statistical analysis of temporal series [27]. The general statement is that enlarging the time window, in order to average the samples of two considered random variables, gives rise to an asymptotic increase of the correlation coefficient. Moreover, since in our case there is also a reduction of data variances when passing from an hourly to a monthly basis, this effect is further amplified due to inverse relation between the variance and the correlation coefficient.

The relation between log-variance and \( RH_S \) is basically nonlinear and the quadratic best fitting between \( m_{\sigma_n} \) and \( RH_S \), showed by a solid line in the right-bottom panel, yields
marzano and d’auria: model-based prediction of amplitude scintillation variance

\[
m_{\sigma_T} = -12.3471 + 0.1079 R_H S - 0.00011 R_H^2.
\]

Note that \( R_H S \) results negatively correlated both with \( \ln(\sigma_T^2) \) and \( T_S \). This mutual behavior in clear air is related to the strong influence of temperature on scintillation, and indirectly on relative humidity and wind velocity [28]. Due to the solar heating of ground, a layer of warm air results at the earth’s surface so that increasing the surface air temperature and producing instability and scintillation. In these cases (and neglecting the impact of humid air masses), there is generally a decrease in relative humidity due to the warmer air being able to hold more moisture (thus increasing the saturated water-vapor pressure). Indeed, after solar heating, the absolute air humidity tends to increase, as shown in [9].

On the top-right panel we also show the linear relationship between \( m_{\sigma_T} \) and \( T_S \), reported by Ortgies in [12] and indicated by a dashed line. For comparison, the solid line on the same plot indicates the linear best fitting of simulated data (given by \( m_{\sigma_T} = -11.9419 + 0.1202 T_S \)). The discrepancy may be possibly attributed to different climate conditions used to derive the two relationships.

Fig. 7 shows the same as Fig. 6, but for normalized log-variance against surface wet refractivity \( N_{W S} \) (calculated as indicated in [10]) and vertically integrated water vapor content \( V_C \). The (linear) correlation between log-variance and \( N_{W S} \) goes from 0.44 on an hourly basis to 0.82 on a monthly basis, while the correlation between log-variance and \( V_C \) goes from 0.44 to 0.82. It is interesting to observe that the correlation \( N_{W S} - T_S \) and \( V_C - T_S \) are 0.81 and 0.78 on an hourly basis and 0.94 and 0.94 on a monthly basis, respectively. From this figure emerges the importance of knowing \( V_C \) to estimate the mean scintillation variance, as will be pointed out in Section V. The quadratic best fitting between \( m_{\sigma_T} \) and \( V_C \), showed by a solid line in the right-bottom panel, yields

\[
m_{\sigma_T} = -13.5103 + 0.3083 V_C - 0.0027 V_C^2.
\]

Again, on the top-right panel we show the linear relationship between \( m_{\sigma_T} \) and \( N_{W S} \) reported by Ortgies in [12] and indicated by a dashed line. For comparison, the solid line on the same plot indicates the quadratic best fitting of simulated data (given by \( m_{\sigma_T} = -13.9393 + 0.1059 N_{W S} - 0.0006 N_{W S}^2 \)). The Ortgies predicted values are lower than those estimated by the best-fitting curve (note that the correction due to the quadratic term \( N_{W S}^2 \) is very small).

### B. Multivariate Estimation of Scintillation Parameters

In order to estimate \( \ln(\sigma_T^2) \) on an hourly basis and \( m_{\sigma_T} \) on a monthly basis, we have tested several regression models both linear and quadratic, selecting as predictors various combination of \( T_S \), \( R_H S \), and \( N_{W S} \). We have also tried a stepwise regression, but without significant improvements with respect to global regression results. As a simulated test data set, we have used one-fifth (731 samples) of the whole ten-year data set on both an hourly and a monthly basis, using the remaining observations (2924 samples) as a training data set.

The estimation results of the simulated test are shown in Table I in terms of root mean square (rms) error, correlation coefficient, and gain ratio. The last quantity expresses the ratio between the training-set standard deviation and the estimate rms error; thus, it should be as much greater than one as possible to select the best estimator. With the aim of making the test more realistic (and robust to noise), we have added to the simulated test measurements a Gaussian noise of zero mean and standard deviation equal to 1% of the value itself. Note that we have not included the mixed terms in the quadratic form.

The analysis of Table I results shows that the best regressive estimators are the one based on surface temperature and humidity in quadratic form (STH2), and the one based on surface temperature and wet refractivity in quadratic form (STN2). It is worth mentioning that most of the prediction algorithms proposed in literature give formulas based on either temperature or wet-refractivity in linear form [6], [10], [12]. Table I shows that a linear relationship is actually proper only for temperature-based predictors, while quadratic estimators using mix meteorological data generally have better performances. The STH2 and STN2 methods have the following expression on an hourly basis:

\[
\langle \sigma_T^2 \rangle_n = \exp(a_{01} + a_{11} T_S + a_{21} R_H S + a_{31} T_S^2 + a_{41} R_H S^2) \tag{18a}
\]

\[
\langle \sigma_T^2 \rangle_n = \exp(a_{02} + a_{12} T_S + a_{22} N_{W S} + a_{32} T_S^2 + a_{42} N_{W S}^2) \tag{18b}
\]

### Table I

<table>
<thead>
<tr>
<th>Predictors</th>
<th>rms error</th>
<th>correlation</th>
<th>gain ratio</th>
</tr>
</thead>
<tbody>
<tr>
<td>( T_S ) linear</td>
<td>1.0767</td>
<td>0.5616</td>
<td>1.1956</td>
</tr>
<tr>
<td>( N_{W S} ) linear</td>
<td>1.0152</td>
<td>0.5717</td>
<td>1.1953</td>
</tr>
<tr>
<td>( T_S ) linear</td>
<td>1.0773</td>
<td>0.5619</td>
<td>1.1949</td>
</tr>
<tr>
<td>( N_{W S} ) linear</td>
<td>1.0152</td>
<td>0.5717</td>
<td>1.1953</td>
</tr>
<tr>
<td>( T_S ) quadratic</td>
<td>1.0773</td>
<td>0.5619</td>
<td>1.1949</td>
</tr>
<tr>
<td>( N_{W S} ) quadratic</td>
<td>1.0152</td>
<td>0.5717</td>
<td>1.1953</td>
</tr>
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<td>( T_S ) quadratic</td>
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</tr>
<tr>
<td>( N_{W S} ) quadratic</td>
<td>1.0152</td>
<td>0.5717</td>
<td>1.1953</td>
</tr>
<tr>
<td>( T_S ) quadratic</td>
<td>1.0773</td>
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<td>1.1949</td>
</tr>
<tr>
<td>( N_{W S} ) quadratic</td>
<td>1.0152</td>
<td>0.5717</td>
<td>1.1953</td>
</tr>
</tbody>
</table>
while on a monthly basis we have

\[ m_{cn} = b_{01} + b_{11} T_s + b_{21} RH_s + b_{31} T_s^2 + b_{41} RH_s^2 \]  

(19a)

\[ m_{cn} = b_{02} + b_{12} T_s + b_{22} N_{WS} + b_{32} T_s^2 + b_{42} N_{WS}^2 \]  

(19b)

The form of (18) depends on the fact that (as already said) \( \ln(\sigma_{n}^2) \) has resulted more correlated to each predictor set than \( \sigma_{n}^2 \) on an hourly basis. Table II reports the regression coefficients of (18) and (19), using the whole ten-year data set on an hourly and a monthly basis. These regression coefficients could be specified for any classification of the RAOB data set (e.g., results on a weekly basis are not shown). The above coefficients are valid for any link specifications, when transforming normalized variables through (16) and (17).

Using the simulated test data set on an hourly and a monthly basis, it results that the major differences between STH2 and STN2 algorithms are relative to an hourly basis case where STN2 tends to underestimate high values of \( \sigma_{n}^2 \), on a monthly basis, the results are fairly good for both methods.

Finally, it can be interesting to provide also the statistical relationships to estimate the vertical average of \( \sigma_{n}^2 \) and the turbulence effective height \( H_{te} \). Performing tests similar to those described above, we have found the following relationships on an hourly basis:

\[
\langle \sigma_{n}^2 \rangle_{va} = \exp(c_{01} + c_{11} T_s + c_{21} RH_s + c_{31} T_s^2 + c_{41} RH_s^2) 
\]

(20a)

\[
\langle \sigma_{n}^2 \rangle_{va} = \exp(c_{02} + c_{12} T_s + c_{22} N_{WS} + c_{32} T_s^2 + c_{42} N_{WS}^2) 
\]

(20b)

and

\[ H_{te} = d_{01} + d_{11} T_s + d_{21} RH_s + d_{31} T_s^2 + d_{41} RH_s^2 \]  

(21a)

\[ H_{te} = d_{02} + d_{12} T_s + d_{22} N_{WS} + d_{32} T_s^2 + d_{42} N_{WS}^2 \]  

(21b)

while on a monthly basis we have

\[ m_{cn} = c_{01} + c_{11} T_s + c_{21} RH_s + c_{31} T_s^2 + c_{41} RH_s^2 \]  

(22a)

\[ m_{cn} = c_{02} + c_{12} T_s + c_{22} N_{WS} + c_{32} T_s^2 + c_{42} N_{WS}^2 \]  

(22b)

and

\[ \mu_{H_{te}} = g_{01} + g_{11} T_s + g_{21} RH_s + g_{31} T_s^2 + g_{41} RH_s^2 \]  

(23a)

\[ \mu_{H_{te}} = g_{02} + g_{12} T_s + g_{22} N_{WS} + g_{32} T_s^2 + g_{42} N_{WS}^2 \]  

(23b)

where \( m_{CN} \) and \( \mu_{H_{te}} \) are the long-term average (on a monthly basis) of \( \ln(\sigma_{n}^2)_{va} \) and \( H_{te} \), respectively.

Table III reports the regression coefficients of (20)–(23), using the whole ten-year data set on an hourly and a monthly basis. On an hourly basis both the estimation of \( \ln(\sigma_{n}^2)_{va} \) and \( H_{te} \) exhibits a poor correlation, which significantly improves on a monthly basis, especially for \( H_{te} \).

### C. Comparison with Olympus Satellite Measurements

In order to validate the above estimation methods, a statistical comparison with Olympus scintillation measurements at 19.8 GHz (namely, 19.77 GHz), acquired at the receiving ground-station in Milan at 30.6° elevation over a period going from July to December 1992, has been performed. The temporal series of scintillation amplitude variance has been provided every one minute after a high-pass filtering of the raw copolar received signal. Clear-air measurements have been carefully selected both by visual inspection and by a threshold on the measured copolar attenuation. Surface meteorological measurements have been available during the Olympus experiment every ten minutes. In order to create a correspondence with RAOB launch time, Olympus measurements around midday and midnight have been screened making an average of 15 min after midday and midnight to match the time needed for the RAOB balloon to complete the ascending path.

Fig. 8 shows the scatterplot between the log-variance of 19.8-GHz Olympus measurements and the log-variance, estimated from (18) and (19) using the simulated test data set on an hourly basis (left panels) and a monthly basis (right panels). Top panels show the results using the STH2 algorithms given in (18a) and (19a), while bottom panels show the results using the STN2 algorithms given in (18b) and (19b). On an hourly basis (left panels) we have 119 samples, while on a monthly basis (right panels) we have only six samples. For comparison, we have also indicated on right panels the long-term estimates performed by applying two algorithms of Ortgies, one using the surface temperature (Ortgies-\( T \)) and the other using the surface wet refractivity (Ortgies-\( N \)) as predictors in (17), applying the proper coefficients \( \alpha \) and \( \beta \). The rms error on an hourly basis tends generally to increase with higher values of scintillation log-variance, but strongly to decrease when doing temporal averaging. Quantitatively speaking, on an hourly basis the bias and the rms errors are, respectively, 0.1680 and 0.8827 for STH2 algorithm, 0.2745 and 1.0081 for STN2 algorithm; on a monthly basis, they are −0.0266 and 0.2624 for STH2 algorithm, 0.1086 and 0.7220 for Ortgies-\( T \), and 0.1086 and 0.7220 for Ortgies-\( N \) algorithm, −0.7624 and 0.3736 for Ortgies-\( T \) algorithm. It is worth mentioning that if for STH2 and STN2 algorithms we use the same normalization proposed by Ortgies (i.e., his \( \alpha \) and

### TABLE II

Regression Coefficients of (18) and (19) and Associated RMS Error Given in \( N_p \) for \( \ln(\sigma_{n}^2)_{va} \) and in \( m \) for \( H_{te} \) and Obtained by Using the Ten-Year Training Data Set.

Note that \( T_s \) is expressed in °C, RH_s in % and \( \ln(\sigma_{n}^2)_{va} \) in %.

<table>
<thead>
<tr>
<th>Equation</th>
<th>coef. 0</th>
<th>coef. 1</th>
<th>coef. 2</th>
<th>coef. 3</th>
<th>coef. 4</th>
<th>rms</th>
</tr>
</thead>
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<tr>
<td>(18a)</td>
<td>-16.6605</td>
<td>0.1796</td>
<td>0.1192</td>
<td>-0.0018</td>
<td>-0.0007</td>
<td>1.6635</td>
</tr>
<tr>
<td>(18b)</td>
<td>-13.8717</td>
<td>0.1771</td>
<td>0.0978</td>
<td>-0.0014</td>
<td>-0.0005</td>
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<tr>
<td>(19a)</td>
<td>-14.9928</td>
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<tr>
<td>(19b)</td>
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<td>0.0315</td>
<td>-0.0056</td>
<td>-0.0004</td>
<td>0.5434</td>
</tr>
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</table>

### TABLE III

Regression Coefficients of (20)–(23) and Associated RMS Error Given in \( N_p \) for \( \ln(\sigma_{n}^2)_{va} \) and in \( m \) for \( H_{te} \) and Obtained by Using the Ten-Year Training Data Set.

Note that \( T_s \) is expressed in °C, RH_s in % and \( \ln(\sigma_{n}^2)_{va} \) in %.

<table>
<thead>
<tr>
<th>Equation</th>
<th>coef. 0</th>
<th>coef. 1</th>
<th>coef. 2</th>
<th>coef. 3</th>
<th>coef. 4</th>
<th>rms</th>
</tr>
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<tbody>
<tr>
<td>(20a)</td>
<td>-34.2705</td>
<td>0.0911</td>
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<td>-0.0005</td>
<td>1.8116</td>
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<td>(20b)</td>
<td>-32.5220</td>
<td>0.0973</td>
<td>0.0641</td>
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<td>-0.0006</td>
<td>1.8235</td>
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<tr>
<td>(21a)</td>
<td>0.1625</td>
<td>-0.0200</td>
<td>0.17</td>
<td>-0.1</td>
<td>-0.01</td>
<td>7.56</td>
</tr>
<tr>
<td>(21b)</td>
<td>871.3157</td>
<td>52.8318</td>
<td>-1.1050</td>
<td>0.0766</td>
<td>-0.0468</td>
<td>751.3827</td>
</tr>
<tr>
<td>(22a)</td>
<td>-23.6095</td>
<td>0.0373</td>
<td>0.0676</td>
<td>0.0036</td>
<td>-0.0005</td>
<td>0.5531</td>
</tr>
<tr>
<td>(22b)</td>
<td>31.1591</td>
<td>0.0857</td>
<td>-0.0464</td>
<td>0.0005</td>
<td>-0.0050</td>
<td>6.0150</td>
</tr>
<tr>
<td>(23a)</td>
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<td>0.2</td>
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<td>(23b)</td>
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<td>-0.0940</td>
<td>-0.48</td>
<td>0.0</td>
<td>0.2</td>
<td>209.4</td>
</tr>
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### Notes

- ERROR GIVEN IN \( N_p \) REGRESSION COEFFICIENTS OF (18) AND (19) AND ASSOCIATED RMS ERROR OBTAINED BY USING THE TEN-YEAR TRAINING DATA SET.
- TABLE II
- TABLE III
Fig. 8. Olympus measured scintillation log-variance at 19.8 GHz and 30.6° elevation against the estimated log-variance on an hourly basis (left panels) and on a monthly basis (right panels). Top panels refer to STH2 prediction algorithms, while bottom panels to STN2 algorithms. On right panels, Ortgies's estimates based on temperature (Ortgies-T) and wet refractivity (Ortgies-N) are also indicated [12].

So far, we have dealt with $m_\sigma$, but, as said, the scintillation variance pdf [given in (14)] is also characterized by the standard deviation $\sigma_\sigma$ of $\sigma_\mathcal{N}$. From experimental data (based on 108 months), Ortgies found values ranging from 0.85 and 1.15, not dependent on frequency and meteorological data; thus, he has assumed $\sigma_\sigma = 1.01$ corresponding to the measured mean value [12]. The results of Karasawa et al. [10] cannot be directly compared in terms of $m_\sigma$ since they adopted a $\Gamma$ distribution for scintillation intensity $\langle \sigma_\mathcal{N} \rangle$; the experimental relationship they have proposed is $\sigma_\sigma = 0.32 \mu_\sigma$, where $\mu_\sigma$ and $\sigma_\sigma$ are the long-term mean and standard deviation of $\langle \sigma_\mathcal{N} \rangle$ [2].

The long-term estimation of $\sigma_\sigma$ can be also carried out by using the model-based approach. The top panels of Fig. 9 show the scatter plot of simulated $\sigma_\sigma$ (left panel) and $\sigma_\sigma$ (right panel) against $m_\sigma$ and $\mu_\sigma$, respectively. For comparison, the experimental relationships of Ortgies and Karasawa et al., are also indicated by dashed lines on the respective plots. From model simulation we can derive the following relationships:

\begin{align}
\sigma_\sigma &= \frac{0.0037 \cdot 0.2948}{m_\sigma}, \quad (24a) \\
\mu_\sigma &= -0.0028 + 0.7254 \sigma_\sigma. \quad (24b)
\end{align}

Both (24a) and (24b) give values higher than those predicted by the Ortgies and Karasawa experimental relationships, respectively.

The bottom panels of Fig. 9 show (right panel) the comparison between the simulated and measured $\sigma_\sigma$ as a function of surface temperature and (left panel) the scatterplot between the measured $m_\sigma$ and $\sigma_\sigma$, where the estimate of $\sigma_\sigma$ made by (24a) is also indicated. From both simulation and measurements no particular trends with temperature can be observed, a fact in agreement with what was noted by Ortgies [12]. The mean value of simulated $\sigma_\sigma$ is 1.5323, while that of Olympus measurements is 0.7619, which is below the values observed by Ortgies. The estimate of $\sigma_\sigma$ gives values between 1.12 and 1.75 and tends to overestimate measured $\sigma_\sigma$ values especially for small log-variance mean values $m_\sigma$. This fact may be due to the undersampling of the Olympus-signal temporal series; that is, to the choice to retain only measurements around midday and midnight. Moreover, this choice can also impact simulation results since we have been forced to neglect the daily evolution of meteorological conditions.

V. CORRELATING SCINTILLATION AND BRIGHTNESS TEMPERATURE

A correlation between scintillation and brightness temperature measurements has been already shown in literature by using experimental data [3], [18]. In this section, we tackle a model investigation in order to include in the prediction methods of amplitude scintillation variance also the measurements of integrated water vapor content. To do this, we have restricted our analysis to the year 1989 of the RAOB data set.

A. Examples of Combined Simulations

At microwave frequencies the tropospheric attenuation is mainly caused by the spectral absorption of water vapor, oxygen and cloud liquid water. Under the assumption of local thermodynamic equilibrium and in the absence of scattering, the thermal emission is expressed in terms of equivalent blackbody brightness temperature $T_B$ using the Rayleigh–Jeans approximation [20]. The $T_B$ can be expressed in terms of total attenuation or optical thickness $\tau$ (decibels) using the radiative transfer equation and introducing the mean radiative temper-
ature, which resumes the vertical dependence of atmospheric variables.

In order to simulate the observed $T_B$ by a ground-based microwave radiometer at a given frequency and in a given direction, we have used the Liebe model [29] to characterize the atmospheric absorption. This model gives a detailed description of the spectral molecular absorption of humid air (possibly, with some cloud liquid) in the frequency range from 1 to 1000 GHz as a function of the local temperature, pressure and relative humidity. The radiative transfer equation has been numerically solved by supposing a plane-parallel atmosphere with levels associated to RAOB vertical soundings. At the top boundary, we have assumed an incident radiation equal to cosmic background $T_B$ (about 2.7 K). Since RAOB does not provide cloud liquid water, the radiative transfer algorithm selects its presence at levels where the relative humidity is higher than 90%. However, all RAOB’s presenting possible cloud liquid layers have been neglected since the framework of this study is valid under clear-air conditions.

Fig. 10 shows the scintillation log-variances at 18.7, 39.5, and 49.5 GHz (Italsat frequencies) against the 23.8-GHz brightness temperatures on an hourly basis (left panel) and a monthly basis (right panel), both derived from each RAOB of 1989 using a radiative transfer model and the scintillation model with the same elevation and antenna aperture used for 19.8-GHz Olympus simulation. The (linear) correlation coefficients on an hourly basis are of 0.51 for 18.7 GHz, 0.50 for 39.6 GHz, and 0.50 for 49.5 GHz scintillation log-variance, while on a monthly basis they increase to 0.93, 0.92, and 0.92, respectively. As expected, scintillation log-variances increase with frequency (maximum values goes from $-4.3905$ Np at 18.7 GHz up to $-2.5874$ Np at 49.5 GHz), even though their dynamic range slightly decrease with frequency (from $2.2469$ Np at 18.7 GHz down to $2.1765$ Np at 49.5 GHz). This correlation between scintillation variances and downwelling brightness temperatures has a physical explanation. As already said, the solar heating of ground causes an increase of the surface air temperature and absolute humidity with production of instability and scintillation. Since brightness temperature basically represents the thermal radiation of the atmosphere, it is clearly motived the positive correlation between $\sigma_\chi^2$ and $T_B$.

As expected in clear-air conditions [16], the linear correlation between brightness temperatures at 23.8 GHz and at 31.6 GHz and the integrated water vapor content $V_C$, is very high (0.99 for 23.8 GHz and 0.98 for 31.6 GHz). The 23.8 GHz $T_B$ exhibits much higher values (up to 120 K in summer) than 31.6 GHz $T_B$ (up to 40 K in summer) due to the fact that 23.8-GHz frequency band is more strongly absorbed by water vapor. These results motivate the use of linear statistical methods to estimate $V_C$ (in kg/m$^2$) from $T_B$ measurements (in K) in clear-air, i.e.,

$$V_C = \alpha_1 + \nu_1 T_{B23}$$
$$V_C = \alpha_2 + \nu_2 T_{B31}$$

where $T_{B23}$ and $T_{B31}$ are the brightness temperatures at 23.8 and 31.6 GHz, respectively. On an hourly basis, the regression coefficients of (25a) are given by $\nu_1 = -5.222$ and $\nu_1 = 0.3531$ and those of (25b) by $\nu_2 = -15.9063$ and $\nu_2 = 0.9241$, with an rms error of 0.906 kg/m$^2$ and 1.032 kg/m$^2$, respectively. On a monthly basis, the regression coefficients of (25a) are given by $\nu_1 = -4.9457$ and $\nu_1 = 0.3475$ and those of (25b) by $\nu_2 = -15.6446$ and $\nu_2 = 0.9489$, with an rms error of 0.241 kg/m$^2$ and 0.267 kg/m$^2$, respectively. Due to the small data dispersion and to the linear relationship, the regressions coefficients of (25) on an hourly basis and on a monthly basis are not very different.

The rms error, obtained using the $T_B$ at 31.6 GHz as a predictor, is higher than that using the $T_B$ at 23.8 GHz and it must be also noted that 31.6 GHz $T_B$ is much more sensitive to the cloud liquid–water presence so that in practical applications the errors can be much greater than those expected. In fact, the use of a dual-channels radiometer with a channel around the 22-GHz water-vapor absorption lines and a window-channel around 30–40 GHz is justified by the need of estimating $V_C$ also in presence of cloud liquid water along the path [16]. More general algorithms use both $T_B$ at 23.8 and 31.6 GHz to estimate both integrated water vapor and cloud liquid water contents in order to fulfill the requirement to operate in any weather condition (except precipitation) [17].

B. New Estimation Methods Including Integrated Water Vapor

In the previous paragraph we have demonstrated how integrated water vapor $V_C$ can be derived from $T_B$ measurements. On the other hand, Fig. 7 has shown a good correlation between normalized scintillation log-variance and $V_C$, so that it is interesting to explore the possibility to estimate $\sigma_\chi^2$ from a combination of surface temperature $T_S$, relative humidity $RH_S$ (or wet refractivity $N_W$), and integrated water vapor $V_C$. Thus, by applying the multivariate statistical regression
Among the predictors significantly elevation angle. The comparison results show that the inclusion of parameters have been also extended to include integrated water vapor estimates among the surface statistical predictors, showing the reduction of rms errors when this extension is performed. Even though this model investigation has clearly shown the potential of microwave radiometers for scintillation studies, further experimental validation are needed. Future aspects of this work will include a systematic validation of the obtained results by using Italsat experiment data in the 20–50 GHz frequency range.

**VI. CONCLUSIONS**

The intermittence phenomenon in turbulent atmosphere has been modeled by means of a statistical formulation of the homogeneous turbulence structure constant. First- and second-order statistical moments of the structure constant have been computed and a procedure has been described in order to estimate the mean structure constant and received amplitude scintillation variance from RAOB data in clear-air conditions. A RAOB data set, collected in Milan from 1980 to 1989 has allowed us to infer the statistical distribution of the scintillation mean parameters to be used in the gain budget evaluation of satellite microwave links.

The statistical regression method has been applied to the estimation of scintillation parameters (such as variance, structure constant, and turbulence height) from surface meteorological data, proposing coefficient tables valid on a monthly basis and on an hourly basis. A preliminary validation of estimation methods has been carried out using Olympus satellite measurements at 19.8 GHz acquired at Milan ground-station at 30° elevation angle. The comparison results show that the proposed methods are fairly accurate and can be promising tools also for comparison with Italsat data up to 50 GHz.

The possibility of using the measurements of a dual-channel microwave radiometer has been also investigated through a model analysis. The capability of microwave radiometers to provide a fairly good estimation of integrated water vapor has been further assessed. The estimation methods of scintillation parameters have been also extended to include integrated water vapor estimates among the surface statistical predictors, showing the reduction of rms errors when this extension is performed. Even though this model investigation has clearly shown the potential of microwave radiometers for scintillation studies, further experimental validation are needed. Future aspects of this work will include a systematic validation of the obtained results by using Italsat experiment data in the 20–50 GHz frequency range.

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