Inversion of Spaceborne X-Band Synthetic Aperture Radar Measurements for Precipitation Remote Sensing Over Land

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Abstract—Several spaceborne X-band synthetic aperture radar (X-SAR) systems were launched in 2007, and more will be launched in the current decade. These sensors may significantly augment the sensors that comprise the Global Precipitation Mission (GPM) constellation. X-SAR rainfall measurements may be beneficial particularly over land where rainfall is difficult to measure by means of satellite microwave radiometers. Inversion techniques to quantitatively derive precipitation fields over land at high spatial resolution are developed and illustrated in this paper. These inversion algorithms are the model-oriented statistical (MOS) methodology and the Volterra integral equation (VIE) approach. Simplified rain-cloud models are used to train and test the inversion algorithms by expecting the evaluated error budget. Two case studies, using data obtained from measurements of SAR/X-SAR in 1994 over Bangladesh and the Amazon, are introduced, and retrieved precipitation maps are discussed. Even though no validation of the precipitation estimates was possible, the obtained results are encouraging, showing physically consistent retrieved structures and patterns.

Index Terms—Inversion methodology, microwave modeling, precipitation, retrieval, X-band synthetic aperture radar (X-SAR).

I. INTRODUCTION

CLIMATE modelers need global precipitation measurements because the released latent heat distribution has a profound effect on the performance of such models. Precipitation measurements are also required to facilitate water management strategies by hydrologists and managers of transportation, agricultural, and flood relief agencies. Although precipitation measurements are widely available in technologically advanced countries, the measurement of precipitation over oceans, mountainous terrain, and less developed regions leaves much to be desired [1].

Since the 1980s, much of our understanding of global precipitation has been provided by spaceborne passive microwave radiometers and a combination of microwave and infrared passive measurements (e.g., [2]–[4]). Unfortunately, spaceborne microwave radiometers, even in combination with infrared sensors, have had limited success in retrieving precipitation over land because they rely heavily on the scattering properties of ice in the upper regions of precipitating clouds. Those scattering properties may be poorly related to surface rainfall rates [5]. This limitation can be overcome over land by space-based radars operating at X- or Ku-band. The Ku-band precipitation radar (PR) aboard the Tropical Rainfall Measurement Mission (TRMM) program has provided unique precipitation measurements over land [6]. Mountainous terrain has presented challenges to both ground- and space-based radars. Radar reflectivity measurements from PR are routinely removed within about 1–2 km from mountainous surfaces to avoid ground clutter. If significant shallow precipitation or rain cells smaller than the 4-km horizontal resolution occur along mountain slopes, then such precipitation may be missed by PR. The measurement of light small rain cells may also be impaired by the signal-to-noise ratio floor of the PR.

A new opportunity to measure precipitation from space may be afforded by the forthcoming availability of several X-band synthetic aperture radars (X-SARs). The TerraSAR-X (TSX) was launched on June 15, 2007, by the Deutsches Zentrum f. Luft u. Raumfahrt (DLR), and another X-SAR will be launched by 2008 [7]. The first of four components of the Constellation of Small Satellites for Mediterranean basin Observation (COSMO-SkyMed, CSK) was launched by the Agenzia Spaziale Italiana (ASI) [8] on June 7, 2007. The first of the four satellites was launched by ASI on June 7, 2007. The Israeli Defense Ministry launched yet another X-band SAR, the TecSAR SAR Technology Demonstration Satellite on January 21, 2008.

Spaceborne X-SARs are generally not designed for atmospheric observation. SARs are often considered “all weather” sensors. However, there is relevant theoretical and experimental evidence that X-band radar may be significantly affected by precipitation occurrence within the synthetically scanned area [9]–[13]. As a matter of fact, PR was designed at Ku-band which is only 4 GHz away from the X-band. Several authors showed that X-SARs are more sensitive to rainfall effects than SARs operating at longer wavelengths, such as L- and C-bands [10]–[13]. For example, this was demonstrated by the Shuttle Missions STS-59 and -68 of 1994 and the STS-99 Shuttle Radar Topography Mission of 2000 carrying the first X-SAR along with the L- and C-band SARs. Rainfall reflectivity at X-band may be enhanced by about 12 dB, and the attenuation increased by about 4 dB when compared with the C-band reflectivity and attenuation [7].

The potential of X-SAR for precipitation retrieval is intriguing. They will probably be able to measure rainfall over...
land with greater sensitivity than from radiometers. The high spatial resolution (less than 100 m) of X-SARs can provide new insights into the structure of precipitating clouds with respect to PR and its future upgrades. X-SAR platforms could also significantly enhance the planned constellation of satellites carrying microwave radiometers and radars that will be part of the foreseen Global Precipitation Measurement mission. These X-SAR satellites, then, may make a valuable contribution to our understanding of the hydrological cycle.

This paper is devoted to the exploration of the potential of spaceborne X-SAR to estimate rainfall over land from both a model and retrieval point of view. The main objective is to provide a framework for a physically based inversion of X-SAR measurements over land. Previous works have demonstrated X-SAR potential for rainfall retrievals, but only recently have there been systematic approaches to design quantitative inversion algorithms [13]–[15]. We will concentrate on X-SAR inversion over land in order to avoid the ambiguities of X-SAR response over ocean in the presence of rainfall [12] and because the hydrological application seems to be very promising, as mentioned.

This paper is organized as follows. In Section II, we will describe the forward model of SAR response to precipitation. In Section III, the inversion methodologies will be extensively illustrated. In Section IV, an application to X-SAR data, which are collected during the SIR-C/X-SAR mission in 1994, will be discussed with the conclusions gathered in Section V.

II. FORWARD MODEL OF X-SAR RESPONSE

In the presence of rainfall, the measured normalized radar cross section (NRCS) signal $\sigma_{\text{SAR}}$ is composed of two components, namely, the backscattering section $\sigma_{\text{surf}}$ from the surface and the volume backscattering $\sigma_{\text{vol}}$ from the precipitation. The total SAR cross-sectional coefficient $\sigma_{\text{SAR}}$ is given by [12], [13]

$$\sigma_{\text{SAR}}(x, y) = \sigma_{\text{surf}}(x, y) + \sigma_{\text{vol}}(x, y)$$

where $x$ is the cross-track direction, $y$ is the along-track direction in a Cartesian coordinate system at ground, and $z$ is the altitude as in Fig. 1. The physical and electromagnetic meaning of (1) will be discussed in the next sections.

We will concentrate on the cross-track sections $x-z$, assuming that the along-track resolution $\Delta y$ has been achieved by means of a proper SAR processing algorithm. By neglecting the so-called range migration, it has been shown that, in the presence of rain, we can estimate $\Delta y \approx \sigma_v (2 \cdot r/v_{\text{sat}})$, where $\sigma_v$ is the hydrometeor velocity standard deviation, $r$ is the slant range from the satellite, and $v_{\text{sat}}$ is the satellite velocity [9]. By assuming that $r = z_{\text{sat}}/\cos \theta$ with an off-nadir angle $\theta = 30^\circ$, a satellite height $z_{\text{sat}} = 600$ km, $v_{\text{sat}} = 7$ km/s, and $\sigma_v = 1$ m/s, the estimated along-track resolution $\Delta y \approx 264$ m. This resolution is poorer than that achievable for a fixed target on the order of few tens of meters. However, this degraded along-track resolution is more than adequate for hydrological purposes. This is particularly true when it is compared with the resolution of spaceborne real aperture radars and microwave radiometers, which may be on the order of several kilometers.

A. Precipitation Cross-Sectional Models

Nadir-pointing radars measure the vertical profile of precipitation; however, we need to model the atmospheric cross section in the $x-z$ plane for oblique viewing SARs. To keep the problem simple, we have assumed the spatial factorization of the precipitation rate $R(x, z)$ (in millimeters per hour) as

$$R(x, z) = H(x) V(z)$$

where $V(z)$ is the vertical distribution of rainfall, and $H(x)$ represents the horizontal variation of $V(z)$.

The variability of $V(z)$ (in millimeters per hour) may be quite high due to cloud microphysics and dynamical evolution. In this paper, a parameterized convection model of $V(z)$ has been built based on isolpeths of equal probabilities of occurrence obtained from measured contoured frequency by altitude

Fig. 1. Schematic view of the SAR response model due to a liquid and ice precipitation cross section $R(x, z)$ under the approximation of plane-wave incidence and a flat surface. The rectangular shape of the rain cloud is chosen for simplicity of representation. The rain-cloud top and the freezing level are at $z_h$ and at $z_0$, respectively, whereas the cloud width $w$ is extending between $x_L$ (left corner) and $x_R$ (right corner). Satellite direction of flight is along the $y$-axis (entering the page), and mean slant off-nadir angle is $\theta$. $x_I$, $x_T$, and $x_K$ are the integration coordinates with different origins. Other symbols are explained in the text.
diagrams [16]. An analytic approximation was fitted to these contours [14]

\[ V(z) = V(0) \left( 0.85 + 0.15 \left( \frac{z_0 - z}{z_0} \right)^{p_L} \right), \quad \text{for} \quad 0 \leq z \leq z_0 \]  

\[ V(z) = V(z_0) \left( \frac{z_h - z}{z_h - z_0} \right)^{p_s}, \quad \text{for} \quad z_0 < z \leq z_h \]  

where \( z_0 \) is the height of the freezing level, and \( z_h \) is the height of the top of the frozen precipitation (see Fig. 1). \( z_0 \) can be obtained from climatology or from temperature soundings. The parameters \( p_L \) and \( p_s \) define the rate at which the liquid and frozen hydrometeor (equivalent) precipitation rates diminish, respectively, with height. Typical values are 0.62 for \( p_L \) and 0.50 for \( p_s \) for equatorial convection.

The horizontal distribution of \( R \) is strongly related to the development of the mesoscale convective system. We have defined a normalized \( H \) function (between zero and one) as

\[ H(x) = \text{shape}(x; w, d_w, t_w, s_w) \]  

where shape is a nondimensional function representing a single rain-cloud cell. The latter can be either a uniform or a trapezoidal shape (defined by the width \( w \) and the minor base \( d_w \), respectively) or an exponential or a Gaussian shape (defined by a scale \( t_w \) and a standard deviation \( s_w \), respectively). In case of a rain-cloud cluster, several cells can be created by superimposing several basic cell shapes. Fig. 2 shows an example of a Gaussian-shaped rain cloud with a typical monsoon-storm vertical profile \( V(z) \), which is characterized by \( V(0) \) equal to about 150 mm/h, \( p_L = 0.62 \), \( p_s = 0.50 \), \( z_0 = 5 \) km, and \( z_h = 14 \) km [17]. In the same figure, the Gaussian shape function \( H(x) \) is also shown with a standard width \( s_w \) of about 5 km.

### B. X-SAR Response Model Due to Precipitation

In order to model the SAR response, for simplicity, we can resort to a plane-wave incidence approximation, as schematically shown in Fig. 1. A rectilinear coordinate \( l-t \) in the cross-track plane \( x-z \) may be defined such that \( l \) is the longitudinal coordinate along the plane-wave direction and \( t \) is the transverse coordinate with respect to \( l \). The SAR returns are computed at each \( x \) position, which is the position of the incidence of each plane-wave direction on the surface. Thus, for a given \( y \) and resolution \( \Delta y \), we can express (1) along the cross-track axis \( x \) as

\[ \sigma_{\text{sr}}(x) = \sigma^0(x) L^2 [\Delta l(x)] \]  

\[ \sigma_{\text{vol}}(x) = \sin \theta \int_{\Delta l(x)} \eta(t) L^2 [\Delta l(t)] dt \]  

where \( \sigma^0 \) is the surface radar cross-sectional coefficient, \( \eta \) is the radar volume reflectivity, and \( L \) is the one-way atmospheric loss factor (dimensionless quantity between zero and one), whereas \( \Delta l \) and \( \Delta t \) are the longitudinal (radial) and transverse path increments along \( l \) and \( t \), respectively. The scattering volume occurs within a slice of oblique thickness \( \Delta r \) in the direction of propagation (see Fig. 1). We can evaluate the error due to the plane-wave approximation compared with the spherical geometric case. This error \( \delta(t) \) can be approximated as \( \delta(t) \approx \Delta l^2/(2 \cdot r_{\text{sat}}) \), where \( \Delta t = z_h/\sin \theta \), and \( r_{\text{sat}} = z_{\text{sat}}/\cos \theta \). For \( \theta = 30^\circ \), \( z_h \approx 10 \) km, and \( z_{\text{sat}} = 600 \) km, the resulting \( \delta(t) \approx 0.2 \) km. This error is within an acceptable spatial resolution for our purposes.
By noting from Fig. 1 that \( t = z / \sin \theta, l = x / \sin \theta, \) and 
\[ \theta \rightarrow \frac{x + z_x}{\tan \theta}, \]
the increase of the NRCS to the left of the cloud, from a shaped cell system of Fig. 2 is shown in Fig. 3. The NRCS scattering behavior (see [14] and [18] for details). The overall NRCS approach is schematically shown in Fig. 4, and each step is briefly illustrated in the following.

\[ \sigma_{\text{eff}}(x) = \sigma^0(x) \exp \left[ -2 \int_{x - z_x \tan \theta}^{x + z_x / \tan \theta} k(x_t, x_t / \tan \theta) \frac{dx_t}{\sin \theta} \right] \]

(6a)

\[ \sigma_{\text{vol}}(x) = \sin \theta \int_{x - X(x, \theta)}^{x} \left[ \eta(x_t, x_t \tan \theta) \times \exp \left[ -2 \int_{x - X(x, \theta)}^{x_t} k(x_k, z_h - x_t / \tan \theta) \frac{dx_k}{\sin \theta} \right] \right] dx_t \cos \theta \]

(6b)

where \( x_t, x_k, \) and \( x_t \) are the integration variables along the \( x \)-direction corresponding to variations along the longitudinal \( x_t(x, x_k) \) and transverse directions \( x_t \). In (6), the integration along \( x_t, x_k, \) and \( x_t \) is assumed to start at \( x - z_0 \tan \theta, x, \) and \( x - X(x, \theta) \), respectively, and it holds that \( X(x, \theta) = (z_n - x_0 \tan \theta) \tan \theta \) (see Fig. 1). It is worth noting in the following statements that: 1) the parameters \( \eta \) and \( k \) depend, in general, on both \( x \) and \( z \); 2) the cross section of the underlying surface \( \sigma^0(x) \) is assumed to be the same as that for the neighboring region; and 3) the angle \( \theta \) should actually be the incidence angle at the surface and not the angle from nadir. These angles are identical in a flat terrain model, as assumed here, but they may differ in reality where the incidence angle changes across the SAR swath.

The relationship between the microwave main parameters \( \eta(x, z) \) and \( k(x, z) \) and the precipitation rate \( R(x, z) \) significantly depends on particle size distribution, shape, and composition. In this paper, we have avoided modeling this microphysical parameterization by resorting to model-oriented regression formulas having the form of a power law

\[
\begin{align*}
\eta(x, z) &= a[R(x, z)]^b \\
k(x, z) &= c[R(x, z)]^d = \frac{\pi^6[K^2]{Z_e(x, z)}}{X^4} \left( e'[R(x, z)]^\theta \right)
\end{align*}
\]

where \( Z_e \) is the Rayleigh equivalent reflectivity factor, \( K \) is the particle dielectric complex factor (with \( |K|^2 = \text{constant} \approx 0.93 \) for water and 0.19 for ice), and \( a, b, c', \) and \( d' \) are usually empirical coefficients depending on the wavelength and precipitation regime. Their typical values for continental convective rain clouds at X-band are as follows (when expressing, \( Z_e \) is in megamillimeters per cubic meter, \( K \) is in decibels per kilometer, and \( R \) is in millimeters per hour): \( a = 2.6 \times 10^{-3}, b = 1.11, c' = 300, \) and \( d' = 1.35 \) for rain and \( a = 5.6 \times 10^{-5}, b = 1.60, c' = 182, \) and \( d' = 1.60 \) for snow with predominant scattering behavior (see [14] and [18] for details).

An example of the X-SAR response due to the Gaussian-shaped cell system of Fig. 2 is shown in Fig. 3. The NRCS behavior can be better understood if we also refer to Fig. 1. The increase of the NRCS to the left of the cloud, from \(-7 \) to \(-4 \) dB, depends only on the ice content at the top of the cloud, which is a result of the oblique projection of the vertical distribution of frozen hydrometers (incidence points between \( x_t \) and \( x_0 \) in Fig. 1). The NRCS to the right ranging from about \(-12 \) to \(-40 \) dB depends mainly on the slant path attenuation by rain (incidence points between \( x_0 \) and \( x_R \) in Fig. 1). The transition regions between \( x_0 \) and \( x_L \) (from pure snow to rain echoes) and between \( x_R \) and \( x_f \) (from rain to snow echoes) explain the observed NRCS and its increase/decrease rate.

### III. INVERSION OF X-SAR MEASUREMENTS

The solution to the X-SAR radar equation (6) requires the extraction of path-integrated quantities. From a mathematical perspective, X-SAR rainfall retrievals are somewhat similar to microwave radiometric rainfall retrievals where the brightness temperature is a weighted integral of precipitation emission and the range resolution is somewhat achieved by means of multifrequency diversity.

In this paper, we have considered two different inversion approaches: the MOS method and the VIE method. Both methodologies will be illustrated in this section.

#### A. MOS Inversion Method

The MOS method builds upon the concepts introduced in [14]. The philosophy behind MOS is to exploit the basic features of NRCS profile behavior and to first identify the horizontal function \( H(x) \). Then, after classifying the shape, the procedure is aimed to retrieve the vertical profile \( V(z) \) from the parameterized models given in (2).

The overall MOS approach is schematically shown in Fig. 4, and each step is briefly illustrated in the following.

1) **Training Database of Simulated Rain Clouds and NRCS:**

For each cell shape \( H(x) \), 50 possible NRCS responses are produced, assuming the cell width (4–12 km), the vertical central profile \( V(z) \), and the top-cloud layer \( z_t \) (10–13 km) as driving variables. The main X-SAR parameters (central frequency, mean off-nadir angle, and along-track ground resolution) are set up, and the surface background value \( \sigma^0 \) is fixed (between \(-7 \) and \(-12 \) dB in this paper). For each class of cell shape and their associated NRCS horizontal profiles \( \sigma_{\text{SAR}}(x) \), a statistical analysis is provided to evaluate the following 11 simulated parameters which are the components of a vector of SAR observations \( x_{\text{SAR}} \):

\[ \begin{align*}
\mu &= \frac{1}{N} \sum_{i=1}^{N} \Delta \sigma_{\text{SAR}}(i) = \frac{1}{N} \sum_{i=1}^{N} \left[ \sigma_{\text{SAR}}(i) - \sigma^0(i) \right] \\
\sigma^2 &= \sqrt{\frac{1}{N-1} \sum_{i=1}^{N} \left[ \Delta \sigma_{\text{SAR}}(i) - \mu \right]^2} \\
\alpha &= \frac{1}{N-1} \sum_{i=1}^{N} \left[ \Delta \sigma_{\text{SAR}}(i) - \mu \right]^{1/2} \frac{1}{\sigma^2} \\
\kappa &= \frac{1}{N-1} \sum_{i=1}^{N} \left[ \Delta \sigma_{\text{SAR}}(i) - \mu \right]^{1/2} \frac{1}{\sigma^2}
\end{align*} \]

computed separately for positive and negative values of the surface-referenced \( \Delta \sigma_{\text{SAR}}(x) \);
2) three numerical gradients as a measure of the $\sigma_{\text{SAR}}$ range-profile slopes around the first inversion of $\sigma_{\text{SAR}}(x)$ at the crossing node $x_0$, its decrease between $x_0$ and $x_{\text{min}}$, and its minimum at $x_{\text{min}}$, where the crossing node $x_0$ is the coordinate at which the input signal crosses the threshold at the background value (see Figs. 1 and 3), and $x_{\text{min}}$ is the $x$ coordinate where $\sigma_{\text{SAR}}(x)$ has one or more minima (see Figs. 1 and 3).

The simulated statistics of these 11 parameters for each shape class $c_H$ is then summarized in terms of the mean vector (centroids) $m_{c_{\text{SAR}}}$ and the covariance matrix $C_{c_{\text{SAR}}}$ of the observable vector $x_{\text{SAR}}$.

2) Preprocessing of the X-SAR Cross-Track Signal: The NRCS measured signal $\sigma_{\text{SAR}}(x)$ is first scanned “pixel-by-pixel” by the algorithm to evaluate the following:

1) the possibility of multiple points of background crossing of $\sigma_{\text{SAR}}(x)$, which can be symptomatic of the presence of a cluster of clouds;
2) the position of the crossing node $x_0$ and the minimum $x_{\text{min}}$, which are previously defined (see Figs. 1 and 3).

3) Classification of the Horizontal Shape Function $H(x)$: The identification of the cell shape is carried out statistically, based upon the maximum-likelihood metric concept. The latter allows us to compare the statistics brought in the vector...
of measured observables \( x_{\text{mSAR}} \) with the one obtained from the simulated data set, expressed by \( m_{\text{cSAR}} \) and \( C_{\text{cSAR}} \), by minimizing the following likelihood distance \( d_{\text{H}} \):

\[
d(c_{\text{H}}) = (x_{\text{mSAR}} - m_{\text{cSAR}})^T C_{\text{cSAR}} (x_{\text{mSAR}} - m_{\text{cSAR}})
\]

where \( c_{\text{H}} \) is the rain-cloud shape class, and \( x_{\text{mSAR}} \) embraces all of the 11 measured statistics for each shape profile. This statistical approach has shown better results than an identification based upon a simple threshold mechanism as it emerges from the classification error analysis.

4) Estimation of Rain-Cloud Width: The locations of \( x_0 \) and \( x_{\text{min}} \) are used to retrieve the width \( w \) or \( s_w \) of the precipitation cells, depending on \( H(x) \). An effective way to retrieve the cell width is to apply a polynomial regression algorithm (in terms of \( x_{\text{min}} - x_0 \)) by exploiting the simulated training database [14], [15]. This analysis is founded upon the observation that \( x_L \) is a good measure of the cloud left coordinate as it indicates the point at which it starts the descent of the decibel values due to the rainfall attenuation. The aforementioned regression formulas yielded a root-mean-square value of the relative error equal to 8% for the considered simulated cases.

5) Reconstruction of the Mean Vertical Precipitation Profile: This step allows us to first evaluate, from the analysis of the input X-SAR measurement behavior, the horizontally averaged rainfall profile defined as

\[
\langle V_{\text{rain}}(z) \rangle \equiv \langle R_{\text{rain}}(z, x) \rangle = V_{\text{rain}}(z) \frac{1}{w} \int_{x_0}^{x_0 + w} \tilde{H}(x)dx, \quad z \leq \hat{z}_0
\]

where the retrieved quantities at previous steps are designated with circumflexes, and the angle brackets stand for horizontal average. The last term of (10a) is strictly valid for rectangular cell shapes of width \( w \) [see (4)], whereas for other horizontal shapes, the mean ground rainfall may be defined in a similar way.

From Fig. 1 it is clear that, once \( H(x) \) is known, we can determine the extinction by the frozen hydrometeors in the range \( x_L - \hat{z}_0 \tan \theta \leq x \leq x_L - \hat{z}_0 \tan \theta \). On the other hand, NRCS is more affected by rainfall attenuation in the range \( x_L < x < x_L + w \). Thus, by taking into account the inclination of the SAR incident wave, the \( \sigma_{\text{SAR}}(x) \) values belonging to the first range are mostly influenced by the snow content, whereas those falling in the second range are more sensitive to the rain content. By means of a polynomial regression algorithm, we can then retrieve the shape-averaged rainfall \( \langle V_{\text{rain}}(0) \rangle \) at ground from proper integrals of \( \sigma_{\text{SAR}}(x) / \sigma^0 \) [see (14) and [15] for details]. Finally, by extracting \( \hat{V}_{\text{rain}}(0) \) from (10a) and using the parameterized models given in (2), we are able to reconstruct first the rain profile \( V_{\text{rain}}(z) \) up to the freezing-level height \( \hat{z}_0 \)

\[
\hat{V}_{\text{rain}}(z) = \hat{V}_{\text{rain}}(0) \left[ 0.85 + 0.15 \left( \frac{\hat{z}_0 - z}{\hat{z}_0} \right)^{\hat{p}_s} \right], \quad z \leq \hat{z}_0
\]

The next step is represented by the retrieval of the snow-content profile \( V_{\text{snow}}(z) \). By using the model in (3b) and through a polynomial regression method, we can derive

\[
V_{\text{snow}}(z) = V_{\text{rain}}(\hat{z}_0) \left( \frac{\hat{z}_h - z}{\hat{z}_h - \hat{z}_0} \right)^{\hat{p}_s}, \quad \hat{z}_0 \leq z \leq \hat{z}_h
\]

with the power-law coefficient given by

\[
\hat{p}_s = \frac{\hat{V}_{\text{rain}}(z = \hat{z}_0)}{V_{\text{snow}}} - 1
\]

An earlier version of this approach was presented in [13]. The overall error budget analysis of the MOS retrieval approach is described later on.

B. VIE Inversion Method

The X-SAR scattering equation in (1) and (6) can be properly manipulated to show that, for vertically uniform rainfall distributions, it can be rewritten as a linear VIE of the second kind [19]. To this purpose, \( R(x, z) \) may be interpreted as an effective vertically averaged rainfall \( R_e(x) \) and is given by

\[
\langle R(x, z) \rangle \equiv R_e(x), \quad \text{for} \quad z \leq z_0
\]

with \( z_0 = \hat{z}_h \) so that both \( k \) and \( \eta \) in (7) depend on \( x \) only. The analytical derivation of the linear VIE is detailed in the Appendix, and its final form is

\[
O(x) = f(x) + \lambda \int_{x_f}^x K(x, s)O(s)ds
\]

where \( K \) is the kernel function, whereas \( f, x_f, \) and \( \lambda \) are the known parameters (see the Appendix). The unknown function \( O(s) \) in (13) is the two-way opacity function defined as

\[
O(s) = \exp \left[ \frac{2}{\sin \theta} \int_0^s k(x')dx' \right]
\]

The solution of (13) can be accomplished by means of a backward iteration, as discussed in the Appendix.

The VIE approach can also be extended to the vertically inhomogeneous precipitation distribution \( R(x, z) \) by exploiting the parameterized model introduced in (2), which is described...
as follows. This overall approach, which is based on the X-SAR preprocessing steps and the VIE solution, is shown in Fig. 5. The X-SAR preprocessing steps are accomplished in the same way as the MOS approach (see Fig. 4).

The basic idea to extend the VIE solution to a vertically inhomogeneous field is to reduce the problem to an equivalent vertically homogeneous rainfall problem. By analyzing a wide set of $\sigma_{\text{SAR}}$ simulated profiles, which are generated by the NRCS forward model whose cloud-top height $z_h$ is higher than the freezing level $z_0$, it appears that the incident wave in the right half of the cloud does not cover the same path in the snow layer and even in the rain layer. By assuming that the rain-cloud shape function $H(x)$ is symmetric as in (4) so that only half of the cloud is considered, we can deal with a difference SAR signal $\Delta \sigma_{\text{SAR}}(x_c)$, which is defined by the difference (in decibels) between the surface background $\sigma^0(x)$ and the value $\sigma_{\text{SAR}}(x_c)$, computed at $x_c = x_R + z_0 \tan(\theta)$, which is, in the backward direction, the first value of $x$ experiencing rain path attenuation (see Fig. 1).

To check if the compensation procedure must be activated, the distance $x_c = x_L + z_h \tan(\theta)$ is first computed. If $x_c$ is larger than $x_L + w/2$ (i.e., snow is probably present), then $x_c$ is set equal to $x_c = x_R + z_0 \tan(\theta)$, and the compensation procedure is carried out. This latter procedure is accomplished between $x_c$ (where the correction is zero) and $x_d = x_L + z_0 \tan(\theta)$ [where the correction is maximum and equal to $\Delta \sigma_{\text{SAR}}(x_c)$]. In case $x_L + z_0 \tan(\theta)$ falls before the half-cloud width, then the compensation procedure is continued until $x_d = x_L + w/2$.

The difference signal $\Delta \sigma_{\text{SAR}}(x_c)$ can be used to compensate $\sigma_{\text{SAR}}(x)$ for the positions $x$ between $x_c$ and $x_d = x_L + w/2$, corresponding to the first optical ray with no path within the snow, by means of an empirical correction factor $F_c(x)$ in decibels

$$F_c(x) = a_s \frac{\Delta \sigma_{\text{SAR}}(x_c)}{(x_R - x_L)^2} (x - x_c)^2 + b_s \frac{\Delta \sigma_{\text{SAR}}(x_c)}{(x_R - x_L)} (x - x_c)$$

$$+ b_s \frac{\Delta \sigma_{\text{SAR}}(x_c)}{(x_R - x_L)} (x - x_c) \quad \text{for} \quad (x_L + w/2) < x < x_c$$

(15)

where $\Delta \sigma_{\text{SAR}}(x) = \sigma^0(x) - \sigma_{\text{SAR}}(x)$ in decibels, $x$ and $z$ are in kilometers, $x_L$ is the estimate of the left coordinate of the rain cloud, and $x_c = x_R + z_0 \tan(\theta)$ (see Fig. 1). The coefficients $a_s$ and $b_s$ have been empirically determined from the model simulations and are $a_s = 0.8$ and $b_s = -0.2$.

The parabolic compensation $F_c(x)$ in (15) becomes larger than $\Delta \sigma_{\text{SAR}}(x)$ if the optical ray also exits from the rain layer due to the lack of rain path attenuation. One way to account for this is to modify the $\sigma_{\text{SAR}}$ background value so that we can compensate for the snow effect and construct an effective signal $\sigma_{\text{SAR eff}}(x)$ whose variations are due only to an equivalent rainfall slab, which are shown at the bottom of the page, where $\sigma_{\text{SAR eff}}(x)$ is in decibels, $x$ is in kilometers, and the last equation holds only if $(x_c + w/2) < (x_R - z_h \tan \theta)$. In the previous equations, snow path attenuation can be neglected because the compensation factor equalizes it and the change of the background value cancels it, whereas its reflectivity, supposing an incidence angle of about 30$^\circ$, does not affect the $\sigma_{\text{SAR}}(x)$ values relative to the right half of the rain cloud. When the clouds are wide enough in comparison with the height (e.g., a simple criterion might be $|(z_h - z_0)/z_0| < 10\%$), the rays incident in the right half of the precipitating cloud experience the same path in the snow layer so that a compensation of $\sigma_{\text{SAR}}$ is not needed.

After constructing an effective SAR profile $\sigma_{\text{SAR eff}}(x)$, it is possible to apply the VIE approach, which is previously described, using the rain constants and $z_0$ as the height to compute $\hat{\eta}(x)$, $\hat{k}(x)$, and $\hat{R}(x)$. The latter can be interpreted as a mean value of each rain-cloud column at a given position $x$. By using an empirical factor $c_r$, which is again derived from model simulations and variable for each cloud shape, we can derive the precipitation intensity at ground $\hat{V}_{\text{rain}}(0)$ from

$$\hat{V}_{\text{rain}}(0)H(x) = c_r \hat{R}(x).$$

(17)

From the previous estimate of $\hat{V}_{\text{rain}}(0)$, we can reconstruct the entire vertical profile $\hat{V}(z)$ using the regression formulas

$$\hat{V}(z) = \begin{cases} \sigma_{\text{SAR eff}}(x) = \sigma_{\text{SAR}}(x) - \sigma_{\text{SAR}}(x_R + 0.5(z_h + z_0) \tan \theta), & \text{for} \quad x_c < x \leq x_f \\ \sigma_{\text{SAR eff}}(x) = \sigma_{\text{SAR}}(x) - \sigma_{\text{SAR}}(x_c), & \text{for} \quad x_R < x \leq x_c \\ \sigma_{\text{SAR eff}}(x) = \sigma_{\text{SAR}}(x) - F_c(x), & \text{for} \quad (x_R - z_h \tan \theta) < x \leq x_R \\ \sigma_{\text{SAR eff}}(x) = \sigma_{\text{SAR}}(x) - \sigma_{\text{SAR}}(x_L - z_h \tan \theta), & \text{for} \quad x_c + w/2 < x \leq (x_R - z_h \tan \theta) \end{cases}$$

(16)
Finally, the precipitation rate distribution $R(x, z)$ is found from (2) through

$$R(x, z) = c_1 \hat{R}_c(x) \left[ \hat{V}(z)/\hat{V}(0) \right]$$

where the vertical variability is normalized to the surface value due to (17).

### C. Error Analysis on Synthetic Data

By using the synthetic measurements of NRCS derived from the training rain-cloud database assuming an additive Gaussian noise of 1-dB standard deviation, both the MOS and VIE methods have been used to retrieve synthetic horizontal rainfall distributions to evaluate the expected error budget.

Fig. 6 shows the estimate of $R(x, z)$ for the example shown in Figs. 1 and 3, obtained by using the MOS algorithm. A fairly good consistency between the simulated and retrieved fields is noted by comparing the top panels of Figs. 3 and 6. If we define the relative error $\varepsilon$ as the difference between the estimated and “true” $R(x, z)$ normalized to the latter, the error standard deviation $\sigma_\varepsilon$ for this example is shown in the middle panel of Fig. 6. As expected, the larger errors are those related to the proper location of the rain cloud, a fact which is evident from the larger discrepancy at the cloud edges. This effect is confirmed by the analysis of the average of the error standard deviation for all simulated rain-cloud cross sections that were available in the training database, where all errors due to the different cloud shapes were combined.

The same analysis may be extended to the results of the VIE inversion algorithm, as shown in Fig. 7. However, smaller errors at the rain-cloud edges are noted both for the given example and for the overall error statistics. These results are due to the combination of the preprocessing X-SAR steps to retrieve the rain-cloud geometry and the VIE reconstruction deduced from (19).

### IV. Rain Retrievals From X-SAR SIR-C Data

As a preliminary application, we have considered satellite data acquired during the X-SAR/SIR-C mission in 1994, even though no ground validation of rain retrievals was available. The X-SAR/SIR-C mission was a joint space mission, which is organized by NASA, DLR, and ASI to test the X-SAR technology using the Endeavor shuttle platform $12 \times 0.4$ m$^2$. At an orbital altitude of 225 km and a look angle range between $17^\circ$ and $63^\circ$ off-nadir, the ground resolution was about $30 \times 30$ m$^2$ [20].

X-SAR data were provided in the multilook ground-range detected image format, which is described by the CEOS standard. Calibrated backscattering coefficient $\sigma_{\text{SAR}}(x, y)$ was obtained from

$$\sigma_{\text{SAR}}(x, y) = \langle I(x, y) \rangle \frac{\sin(\theta_i - \alpha_{sl})}{M_S \sin(\theta_i)} - \langle N_{\text{raw}} \rangle G_N C_{\text{rad}}(x, y)$$

where $\langle I(x, y) \rangle$ is the average value of pixel power, $M_S$ is a calibration constant, $\theta_i$ is the nominal incidence angle (for our data, it is estimated to be equal to $30^\circ$), $\alpha_{sl}$ is the terrain local slope (taken as $0^\circ$ in agreement with the assumed flat terrain model), $\langle N_{\text{raw}} \rangle$ is the average noise power on raw data, $G_N$ is the processor noise gain, and $C_{\text{rad}}(x, y)$ is the cross-track radiometric correction vector of each pixel.

### A. Case Studies

Two case studies have been considered in this paper within two different geographical areas: the Bangladesh coastline near the Indian ocean and the Amazon forest in Brazil. These two scenes exhibit a quite homogeneous background which fulfills our assumption of a constant $\sigma_0$ within the SAR forward model.

The left panel of Fig. 8 shows the measured X-SAR NRCS calibrated map over Begamganj, Bangladesh. The image was acquired on April 18, 1994, at 5:43:26 GMT (data take ID: 143.52) with latitude/longitude of $23.17^\circ$ North/$91.00^\circ$ East at the image center. The X-SAR image was spatially integrated to a pixel size of about 300 m, and the surface background NRCS $\sigma_0$ was inferred to be about $-10$ dB using areas without atmospheric effects in the same region. The upper right-hand part of this image shows an isolated moderately tall rain cell that scatters little radiation. The rain mainly attenuates radiation on its way to and from the surface. The lower right-hand side of the image shows a distributed mesoscale convective system, which is characterized by a “plume” which appears to be sheared to the left.

The second case study is shown in the right panel of Fig. 8 and refers to the area of Sena Madureira within the Amazon forest, Brazil. The image was acquired on April 15, 1994, at 18:45:50 GMT with the image center at a latitude/longitude at $9.01^\circ$ South/$68.38^\circ$ West. The surface background NRCS $\sigma_0$ was set to $-11.7$ after looking at areas without atmospheric effects. After a spatial averaging to about 300 m, the calibrated image has a size of $8395 \times 2397$ pixels. The serpentine signature of the river Sena is evident in the middle of the image, whereas dark areas seem to indicate the presence of moderately tall rain cells that mainly absorb backscattered radiation.

### B. Retrieval Results

For the Bangladesh case study shown in Fig. 8, Fig. 9 shows the retrieved surface precipitation derived from the MOS and VIE inversion algorithms. Coupled with Fig. 9, Fig. 10 shows the retrieved surface precipitation within the subarea identified by a white rectangle in Fig. 8 derived from the MOS and VIE inversion algorithms. Finally, Fig. 11 shows the retrieved precipitation cross section along the red line in Fig. 8, which is derived from the MOS and VIE algorithms.

The previous figures show that the two algorithms, namely, MOS and VIE, tend to produce mutually consistent results, a feature which was not necessarily expected. A cross-track elongation of the retrieved rainfall patterns is shown in Figs. 9 and 10, which is probably due to the increased estimation errors at the cell edges (as shown in Figs. 6 and 7) and also
Fig. 6. Cross section of the relative error budget of MOS method. (Top panel) Estimate of $R(x,z)$ for the example shown in Figs. 1 and 3. (Middle panel) Relative error standard deviation for the example shown in Figs. 1 and 3. (Bottom panel) Average of the relative error standard deviation for all simulated rain-cloud cross sections.

The same plots of Figs. 9 and 11 are shown in Figs. 12 and 13, but for the Amazon case study. Comments on these figures are similar to those already made for the previous figures. Since the selected X-SAR image is characterized only by absorption signatures, the cross-track section of the retrieved structure provides a rain cell up to the freezing level.
V. CONCLUSION

Unlike space-based near-nadir viewing radars, such as the TRMM-PR which can provide highly resolved vertical precipitation profiles, X-SARs will mainly measure the slant-path integrated scattering and attenuation of precipitation in orthogonal oblique directions. As a consequence, the algorithms to retrieve rainfall distributions from X-SAR data are more convoluted than those used for conventional radars.

A microwave model for simulating the NRCS SAR response was set up in this paper and exploited to train the proposed X-SAR retrieval algorithms. As in all rainfall retrievals from single-frequency radar [6], numerous simplifying assumptions were needed. For example, the surface below the rain cloud was modeled as a flat terrain with a constant $\sigma^0$ (which is the same assumption used in the TRMM-PR surface-reference technique [6]). The size distribution of all
hydrometeors and their microwave behavior was assumed to be known. A sharp transition was imposed between rain and snow at the freezing height, and no graupel or supercooled water hydrometeors were considered. The rain-cloud 2-D shape was modeled as the product of horizontal and vertical distribution functions.

Two inversion approaches were applied to model precipitation distributions, namely, a MOS inversion methodology and a generalized solution of the VIE. The MOS approach exhibits a high flexibility as it is based on forward physical-electromagnetic numerical model used to train a classification scheme and an estimation algorithm. The VIE approach is an elegant way to mathematically treat the SAR forward and inverse problem, but it needs to be generalized to vertically inhomogeneous media to be useful in atmospheric applications.

By using synthetic data, both approaches yielded a relative error budget within 20%, except at the edge of the retrieved rain clouds. Two examples of the MOS and VIE retrievals, which are applied to the NRCS data acquired from the 1994 X-SAR/SIR-C Shuttle Mission, were finally presented and discussed, showing a physically consistent retrieved rain structure and mutually consistent precipitation patterns.

We could not perform any validation of rainfall retrievals as ground-based network data were not available. Future work should be devoted to this crucial issue, focusing on the recently launched X-SAR platforms, namely, TSX and CSK. Moreover, in order to remove some crude assumptions on the precipitation spatial distribution, a more realistic description of hydrometeor water contents can ultimately be included in both retrieval algorithms by employing outputs from
cloud-resolving models, as is done in passive microwave radiometer retrievals [2]. Another important aspect will be the capability to treat rough terrain with variable NCRS surface backgrounds within the X-SAR forward and inverse models in order to consider the local incident angle. The potential of using multifrequency SAR measurements of precipitation, such as at Ku- and Ka-bands, needs to be investigated to reduce the uncertainty of the estimated rainfall parameters.
Fig. 11. Retrieved precipitation cross section along the red line (left panels) and the cyan line (right panels) in Fig. 8 for the Bangladesh case study, which is derived from the MOS (top panel) and VIE (bottom panel) algorithms.

Fig. 12. Same as in Fig. 10, but for the Amazon case study.
APPENDIX
VIE ANALYTICAL FORMULATION

Mathematical details on the formulation of VIE approach are described here, following the analytical framework presented in [19]. For a vertically uniform and horizontally inhomogeneous rainfall distribution $R_e(x)$ between $z = 0$ and $z = z_0 = z_h$, the SAR equation in (5) can be rewritten as

$$\sigma_{SAR}(x) = \sigma_0(x) \exp \left[ -\frac{2}{\sin \theta} \int_x^{x+z_h/\tan \theta} k(x_t)dx_t \right]$$

$$+ \tan \theta \int_x^{x+z_h/\tan \theta} \eta(x_t) dx_t$$

$$\times \exp \left[ -\frac{2}{\sin \theta} \int_x^{x+z_h/\tan \theta} k(x_t)dx_t \right]$$

$$\times \int_x^{x+z_h/\tan \theta} \eta(x_t) dx_t$$

(A.1)

where $X(x, \theta) = (z_h - x_t \tan \theta) \tan \theta$ (see Fig. 1). The attenuation and the reflectivity are

$$\left\{ \begin{array}{l} k(x) = a \left[ R_e(x) \right]^b \\ \eta(x) = c \left[ R_e(x) \right]^d \end{array} \right.$$

(A.2)

where $R_e(x)$ is defined in (17), and the coefficients are those of (7). Once the two-way opacity function $O(x)$ is defined as

$$O(x) = \exp \left[ \frac{2}{\sin \theta} \int_0^x k(\nu)d\nu \right]$$

(A.3)

the rainfall parameters can be easily derived

$$\left\{ \begin{array}{l} k(x) = \frac{O'(x)}{2O(x)} \sin \theta \\ \eta(x) = c \left[ \frac{\sin \theta O'(x)}{2O(x)} \right]^{d/b} \\ R_e(x) = \left[ \frac{\sin \theta O'(x)}{2O(x)} \right]^{1/b} \end{array} \right.$$

(A.4)

where $O'(x)$ is the derivative of $O(x)$. We can transform (A.1) in terms of $O(x)$ as follows:

$$\sigma_{SAR}(x) = \sigma_0(x) \frac{O(x-z_h \tan \theta)}{O(x)}$$

$$+ C \int_x^{x+z_h/\tan \theta} \left[ \frac{O'(x_t)}{O(x_t)} \right]^{d/b} \frac{O(x_t-X(x_t, \theta))}{O(x_t)} dx_t$$

(A.5)
where
\[
C = \tan \theta \left[ c \left( \frac{\sin \theta}{2a} \right)^{d/b} \right]. \tag{A.6}
\]

By shifting the origin from \( x \) to \( x + z_h \cdot \tan \theta \) and introducing the integration variable \( s \) defined as
\[
s = \frac{x_t - x \sin^2 \theta - z_h \tan \theta}{\cos^2 \theta} \tag{A.7}
\]
with \( dx = (\cos \theta)^2 dt \), we can rewrite (A.5) as
\[
\sigma_{\text{SAR}}(x + z_h \tan \theta) = \frac{\sigma^0(x + z_h \tan \theta) O(x)}{O(x + z_h \tan \theta)} + (\cos \theta)^2 C \int_x^{x + z_h / (\sin \theta \cos \theta)} N[x_t(x, s)] O(s) \, ds \tag{A.8}
\]
where
\[
\begin{cases}
N[x_t(x, s)] = \frac{O'(x_s)}{O(x_s)} \left( \frac{d/b}{\cos \theta} \right)
\end{cases}
\]
\[
x_t(x, s) = s \cos^2 \theta + x \sin^2 \theta + z_h \tan \theta.
\]

By formally solving the previous equation (A.8) for \( O(x) \), we can obtain a linear VIE
\[
O(x) = f_0(x) - \lambda \int_x^{x + z_h / (\sin \theta \cos \theta)} K[x_t(x, s)] O(s) \, ds \tag{A.10}
\]
where
\[
\begin{cases}
f_0(x) = \frac{O(x + z_h \tan \theta) \sigma_{\text{SAR}}(x + z_h \tan \theta)}{\sigma^0(x + z_h \tan \theta)}
K(x, s) = \frac{N[x_t(x, s)] O(x + z_h \tan \theta)}{\sigma^0(x + z_h \tan \theta)}
\lambda = (\cos \theta)^2 C.
\end{cases} \tag{A.11}
\]

Let us assume that \( x_f = x_R + z_h \tan \theta \) which is the farthest cross-track point outside the rain cloud such that the incident wave does not experience any atmospheric path attenuation (see Fig. 1 when \( z_h = z_0 \)). In (A.10), the function \( f_0(x) \) and the kernel \( K(x, s) \) depend on the unknown \( O(x) \). However, if \( O(x) \) is known outside the medium boundary for \( x \geq x_f \), then the following can be performed.

1) The function \( f_0(x) \) in (A.11) may be determined for \( x \geq (x_f - z_h \tan \theta) \) using (A.10) since everything is known in that range.

2) The kernel \( K(x, s) \) may be evaluated for \( x \geq (x_f - z_h \tan \theta) \) and \( s \geq (x - z_h \tan \theta / \cos^2 \theta) \) which implies that \( x_f \geq x \), as deduced from (A.11) coupled with (A.9).

3) For \( s \geq x_f \), the integral in (A.10) can be calculated and set as constant when combined with \( f_0(x) \).

Thus, in the interval \( x_f - z_h \tan \theta \leq x \leq x_f \), the VIE in (A.10) can be simplified as
\[
O(x) = f(x) + \frac{x}{x_f} \int_{x_f}^{x} K(x, s) O(s) \, ds \tag{A.12}
\]
where
\[
f(x) = f_0(x) + \frac{x}{x_f} \int_{x_f}^{x + z_h / (\sin \theta \cos \theta)} K(x, s) O(s) \, ds. \tag{A.13}
\]

Equation (A.12) is identical to (13). The linear VIE can be solved iteratively moving backward to the left from the farthest point with respect to the satellite (see Fig. 1), exploiting the information about the finite extension of the rainfall cell. In fact, for \( x \geq x_f \), we know that \( R(x) = 0 \) so that \( k(x) = 0 \), \( O(x) = 1 \) from (A.3), and \( O'(x) = 0 \) from (A.4). This implies that for \( x \geq x_f \), \( N(x_t) = 0 \) and \( K(x, s) = 0 \), and from (A.8), we can evaluate the following ratio:
\[
O(x) = \frac{\sigma_{\text{SAR}}(x + z_h \tan \theta)}{\sigma^0(x + z_h \tan \theta)}, \tag{A.14}
\]
for \( x_f - z_h \tan \theta \leq x \leq x_f \).

Note that, at this step, \( O(x) \) is also known for \( x \geq (x_f - z_h \tan \theta) \). This means that, within the interval \( (x_f - z_h \tan \theta) \leq x \leq (x_f - z_h \tan \theta) \), we can compute the function \( f(x) \) by substituting (A.14) into (A.11) and (A.13)
\[
f(x) = \frac{\sigma_{\text{SAR}}(x + 2z_h \tan \theta) \sigma_{\text{SAR}}(x + z_h \tan \theta)}{\sigma^0(x + 2z_h \tan \theta) \sigma^0(x + z_h \tan \theta)}
+ \frac{x}{x_f - z_h \tan \theta} \int_{x_f - z_h \tan \theta}^{x + z_h / (\sin \theta \cos \theta)} K(x, s) O(s) \, ds \tag{A.15}
\]
where the kernel function \( K(x, s) \) is given by
\[
K(x, s) = \frac{\sigma_{\text{SAR}}(x + 2z_h \tan \theta) N[x_t(x, s)]}{\sigma^0(x + 2z_h \tan \theta) \sigma^0(x + z_h \tan \theta)}. \tag{A.16}
\]

By iterating this numerical process through (A.12)–(A.16) for the successive intervals \( x_f - (n + 1)z_h \tan \theta \leq x \leq x_f - nz_h \tan \theta \) with \( n \) from 1 to \( n_{\text{tot}} \) (i.e., the number of pixels along the cross-track direction \( x \)), we can evaluate the unknown two-way opacity \( O(x) \) over the whole SAR cross-track range. From the knowledge of \( O(x) \), we can then retrieve the effective precipitation rate distribution \( R_e(x) \) and specific attenuation \( k(x) \) from (A.4).
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